REPORT No. 115

BENDING MOMENTS, ENVELOPE, AND CABLE STRESSES IN NON-RIGID AIRSHIPS

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SUMMARY.

No simple but comprehensive method of calculating the principal stresses in the envelope of a non-rigid airship has hitherto been described and published in the English language. The present report describes the theory of the calculations and the methods which are in use in the Bureau of Aeronautics, United States Navy. The principal stresses are due to the gas pressure and the unequal distribution of weight and buoyancy, and the concentrated loads from the car suspension cables.

The second part of the report deals with the variations of tensions in the car suspension cables of any type of airship, with special reference to the rigid type, due to the propeller thrust or the inclination of the airship longitudinally.

INTRODUCTION.

The Third Annual Report (1917) of the National Advisory Committee for Aeronautics contained a translation by Prof. Karl K. Darrow of the report of the investigations by two German engineers, Dr. Rudolf Haas and Alexander Dietzius, into "The Stretching of the Fabric and the Deformation of the Envelope in Non-Rigid Balloons." That report, prepared with characteristic German care and exhaustiveness, included a description of the theory and procedure for finding the stresses in the envelope when the bending moment is known, but failed to describe how the stresses in the car-suspension cables and hence the bending moment due to the weight of the car and its load may be determined when the system of suspension cables is at all complex. The writer was therefore asked by the National Advisory Committee for Aeronautics to prepare a more comprehensive report on the subject.

THEORY AND METHOD OF PROCEDURE.

The airship designer has an infinite choice of possible arrangements of the car-suspension system, and even when the layout of the cables is determined there is an infinite number of ways in which the load may be arbitrarily distributed among them, each distribution producing its own peculiar variation of the bending moment along the envelope, so that without a theory of the methods to be followed in the design of the suspension system but little can be done in the calculation of the stresses in the envelope. The present report is intended to furnish an account of a theory and method of procedure applicable to modern airships in which it is customary to suspend a short car close up to the envelope. When the bending moment is determined in accordance with this procedure, the stresses in the fabric of the envelope due to the bending may be determined to a good order of accuracy by application of the ordinary formula for the fiber stresses in a loaded beam, without recourse to the elaborate refinements and corrections described by Haas and Dietzius.

The hull or envelope of an airship may be regarded as a beam loaded by the forces of buoyancy acting upward or positively and the weights acting downward or negatively. The algebraic sums of these forces and of their moments about any point are zero when the airship is in equilibrium. The buoyancy forces may be determined by the well-known methods of naval architecture from a curve of gross buoyancy. The theory and method of drawing such a curve are given in detail in the standard textbooks of naval architecture (e. g., Atwood's *Theoretical Naval Architecture*), but briefly it may be described as a curve drawn upon a base line representing the length of the ship and having ordinates which represent to a suitable scale the gross buoyancy per unit of length throughout the length of the ship. The total area inclosed by the curve gives to scale the gross buoyancy of the whole airship.

The determination of the downward acting forces is a much more complex matter than of the upward ones, because in addition to the weights on the envelope there are the vertical components of the forces in the car-suspension cables which, as already stated, may be distributed in an infinite number of ways.

The total bending moment in the envelope is not alone due to the vertical forces, and it will be shown that account must be taken of the bending moments resulting from the horizontal components of the suspension forces and also the so-called *gas pressure bending moment* which is due to the increase of gas pressure upward in the envelope, producing longitudinal forces which are greater above than below the neutral axis.

The following methods of assigning the tensions in the suspension cables and of computing the bending moments in the envelope have been devised in the Bureau of Construction and Repair. The reader will probably find that the various steps may be most readily understood by comparing with the text each successive curve or diagram shown in the accompanying figure of the load and bending moment diagram of a non-rigid airship.

The procedure is as follows:

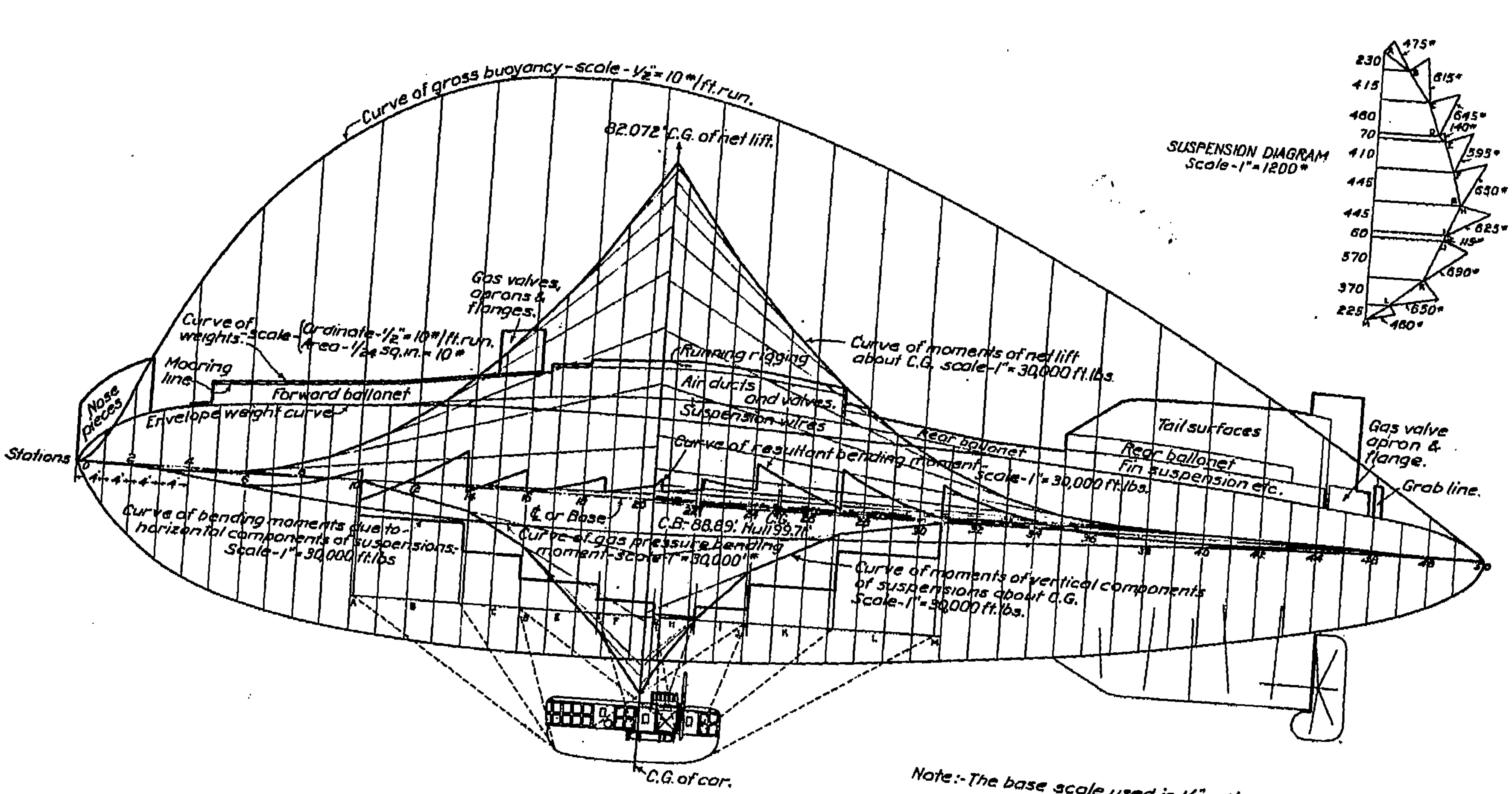
(*a*) A curve of gross buoyancy is drawn. For an envelope having circular cross sections the ordinate at any station is proportional to $k\pi R^2$, where R is the radius of the cross section and k is the lift of the gas per unit volume.

(*b*) A curve of envelope weights is laid off on the same base line and to the same scale as the curve of envelope weights. The area between the curves of gross buoyancy and envelope weights represents the net lift, i. e., the weight of the car and its load.

(*c*) The areas of the net lift curve between successive stations are found by a planimeter, and the longitudinal center of gravity of each of these areas estimated by eye; in most cases the mid-point of the station interval is assumed to be the center of gravity of the area. Moments of these areas are taken about some convenient station in order to calculate the center of gravity of the net lift. If the length of the envelope is divided up into 20 or more stations, this method of obtaining the moments and center of gravity of the area of the net lift curve is sufficiently accurate. The car should be placed so that its center of gravity is vertically under the center of gravity of net lift.

(*d*) The moments of each interval of the net lift curve are next taken about the center of gravity of net lift (referred to hereafter as the C. G.), and a moment curve of net lift drawn as if the envelope were a cantilever held by a single downward force at the C. G. This curve is plotted by laying off successively upon a vertical line through the C. G. the moments about that line due to each interval of the net lift curve, working inward from the bow and stern, and drawing straight lines successively from the points laid off to the previous line at a point vertically above the C. G. of the area for which the line being drawn represents the moment. The construction of this curve may be readily understood by inspection of the curve of moments of net lift about the C. G. in the accompanying load and bending moment diagram for a nonrigid airship.

(*e*) A diagram of the tensions in the car-suspension cables is constructed to fulfill the conditions that the sum of the horizontal components of the tensions is zero, and the sum of the vertical components equals the net lift, and the sum of the moments about the C. G. is zero. These conditions are obviously necessary if the airship is to be at rest in horizontal trim and static equilibrium. To simplify the problem of moments, the points of attachment of the suspensions to the envelope are all assumed to lie on one horizontal line. The necessary condition with regard to moments then becomes that the sum of the moments of the vertical com-



LOAD AND BENDING MOMENT DIAGRAM FOR A NON-RIGID AIRSHIP

FIG. 1.

ponents of the suspension about the C. G. equal zero. It can be seen at once from the suspension diagram whether or not the conditions in regard to the magnitude of the vertical and horizontal components of the suspension forces are fulfilled. The moments may also be obtained graphically, but it is more expeditious to use a combination of analytical and graphical methods, using the suspension force diagram to obtain the vertical component of the tension, and multiplying this component by the horizontal distance of the point of attachment of the suspension cable from the C. G. to obtain the moment. The leads of the suspension cables and the adjustment of the tensions among them are juggled by trial and error until the necessary conditions are fulfilled. The suspension diagram used in these calculations shows only the tensions in the vertical longitudinal plane. To find the actual tensions in the cables it is necessary to make a correction for the inclination of the cables to that plane.

Where there is a multiplicity of suspension cables the number of possible solutions is infinite. Besides the foregoing conditions which must be fulfilled there are others at which the designer should aim. The tensions in the car suspension should be chosen so as to produce as small a bending moment in the envelope as is practicable and consistent with other requirements, and although there is no direct means of knowing precisely what bending moments will result from any given design of suspension until the bending moment diagram is finished, a designer with a little experience in working out diagrams in this manner should have no difficulty in hitting upon a fairly good solution at the first attempt in any given case, and the second attempt should give entirely satisfactory results. Another condition of almost equal importance is that the weight of the car should be taken by wires as nearly vertical as possible, because inclined wires receive heavy loads when the airship inclines up or down to extreme angles, even if their tensions are quite small when the airship is in horizontal trim. These last two conditions may be found to be mutually inconsistent in some cases so that the designer is compelled to compromise. Another desirable condition, but usually of minor importance, is that the bending moments in the car should be small. With the short cars of modern airships this condition is easily realized.

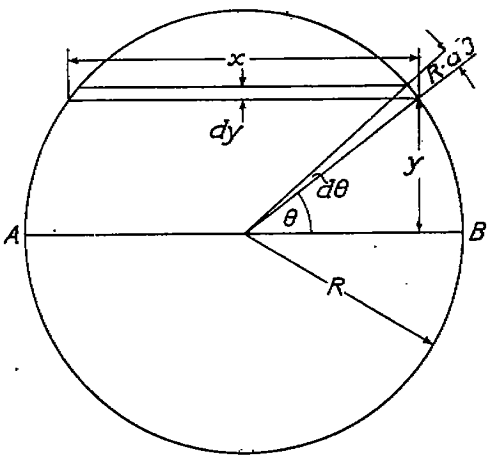


FIG. 2.

(f) A curve of the moments of the vertical components of the car suspension at the points of attachment to the envelope is constructed in a similar manner to the curve of moments of net lift. The line through the attachments of the suspension cables to the envelope is considered to be a cantilever supported by a single upward force at the C. G., and loaded with the vertical components of the suspensions.

(g) A curve is drawn of moments due to the horizontal components of the tensions in the suspension cables multiplied by the distance of the points of attachment below the longitudinal axis of the envelope. The ordinates of this curve are constant between the points of attachment of the suspension cables to the envelope, instead of continuously increasing toward the C. G. as in the curves previously drawn. This bending moment due to the horizontal components of the suspension forces has sometimes been neglected, and the tensions in the cables assigned with regard only to the vertical forces. In such cases the airship is subject to severe negative bending moment, causing the bow and stern to droop. The fact that the horizontal components of the suspension forces are the cause of an important bending moment makes it possible to support a short car close up to the envelope without requiring an excessive tension in the end suspension cables to keep the bow and stern of the airship from curving upward.

(h) A curve of gas pressure bending moments is constructed. This is a negative moment resulting from the increase of gas pressure upward. If, as shown in figure 2, an envelope having circular cross sections of radius R, x = width across the section at a distance y from the horizontal diameter of the section, and k is the lift per unit volume of the gas, the increase in pressure upward produces differences of longitudinal forces on the section causing a moment about the horizontal diameter given by:

$$-M=\int_{-R}^{+R} Kxy^2dy$$

But

$$x=2\sqrt{R^2-y^2}$$

Therefore:

$$-M=\int_{-R}^{+R} 2ky^2\sqrt{R^2-y^2}dy$$

$$=\frac{\pi}{4}kR^4.$$

The curve of gas pressure bending moments is plotted in accordance with this formula.

(*i*) There are now four bending moment curves, a positive one due to the moment of the net lift about the C. G. and three negative ones. A final curve is drawn as the resultant of these four and its ordinates give the net bending moment in the envelope.

The accompanying figure shows the bending moments in a nonrigid airship worked out in accordance with this method. In spite of the fact that the car carries all fuel and is short and close to the envelope, the bending moment is nowhere large.

It was shown by Haas and Dietzius that when the bending moment and the moment of inertia of the cross-section of an airship envelop are known, the longitudinal stresses in the fabric due to bending may be computed by the ordinary beam-bending formula, $f=My/I$. The internal gas pressure produces tension in the envelope, and since fabric is incapable of sustaining compression it is essential that the compression due to bending shall not exceed the tension due to gas pressure.

In dealing with the stress and moment of inertia of an airship envelope it is customary to consider the cross section of the envelope as an infinitely thin ring, so that the stress is expressed as pounds/inch instead of as pounds/inch^2, and the dimensions of the moment of inertia are L^3 instead of L^4. In figure 2 the circle represents the cross section of the envelope of an airship, of radius R, and the moment of inertia about any diameter such as AB is given by:

$$I=4\int_0^{\frac{\pi}{2}} y^2Rd\theta$$

But

$$y=R\sin\theta$$

Therefore

$$I=4R^3\int_0^{\frac{\pi}{2}}\sin^2\theta d\theta$$

$$=\pi R^3.$$

When the bending moment, M, at a circular-cross section is determined, the maximum unit stress in the fabric at that cross section due to the bending alone is when $y=R$, so that the maximum stress is given by:

$$f=\frac{My}{I}=\frac{M}{\pi R^2}.$$

The work of Haas and Dietzius is largely devoted to the methods of ascertaining the degree to which the cross section of the envelope departs from the designed circle. Owing to the stretch of the fabric the circumference increases about 3 per cent with modern three-ply fabrics, and the car suspension causes a distortion of the cross section increasing the height a further 2½ per cent approximately.

The fabric is also reinforced by the overlap at the seams, amounting to about 5 per cent additional strength. The total effect is to make the moment of inertia of the actual cross section about 19 per cent greater than in the designed circular cross section. The increase in the moment of inertia in any particular case may be calculated from the characteristics of the fabric and the stresses due to the gas pressure and the car suspension in accordance with the methods described in detail by Haas and Dietzius, but in ordinary practice the arbitrary correction of 19 per cent is sufficiently accurate.

The longitudinal stresses due to the internal gas pressure and the longitudinal components of the tensions in the car-suspension cables are superimposed on the stresses due to the bending moment.

The total gas pressure force upon any cross section is equal to the area of the section multiplied by the mean gas pressure, i. e., by the pressure at the mid-height of the envelope. The compressive force due to the suspension cables may be found from the suspension diagram. The resultant of these longitudinal forces does not lie along the neutral axis of the envelope, but the effect of this offset position has been taken care of in the methods already described for calculation of the bending moment.

The stress per unit width of fabric due to the longitudinal forces exclusive of bending is obviously equal to the forces divided by the circumference of the section with corrections made for the stretch and the effect of the seams. This stress is superimposed upon that due to the total bending moment.

In tapered portions of the envelope a correction must be applied to the calculation of the longitudinal stress. Let the longitudinal tangent to the envelope be inclined at an angle α to the longitudinal axis. Then the true longitudinal stress is obtained by dividing the apparent longitudinal stress by cos α. The apparent longitudinal stress is calculated by assuming the envelope to be cylindrical at the cross section considered.

Example: Find the total longitudinal stress per unit width of fabric at the top and bottom of an airship envelope at a cross section where the designed radius is 21 feet, the mean gas pressure is 2 inches of water, the total bending moment is $-50,000$ foot-pounds, the longitudinal component of the tensions in the suspension cables is 1,600 pounds, and $\alpha=5°$.

The designed area of the cross section is $21^2\times\pi=1,390$ feet,^{2} and adding 9 per cent for the stretch of the fabric the actual area is 1,515 feet.^{2} The mean gas pressure is 2 inches of water $=10.4$ pounds/ft.^{2} The total longitudinal force is $(1,515\times10.4)-1,600=14,180$ pounds.

The circumference of the cross section is $2\pi\times21=132$ feet, or 136 feet, allowing 3 per cent for the stretch. Neglecting the effect of α for the meantime, but allowing 5 per cent additional strength due to the seams, the unit stress in the fabric due to the total longitudinal force upon the cross section is

$$\frac{14,180}{1.05\times136}=99.3 \text{ lbs./ft.}$$

The moment of inertia of the designed section is $\pi\times21^3=29,200$ ft.,^{3} or $1.19\times29,200=34,800$ ft.,^{3} including the allowances for stretch, deformation, and seams. The distance of the top of the envelope from the neutral axis is $21\times1.03\times1.025=22.2$ ft., including the corrections for stretch and deformation. The unit stress due to the bending is given by:

$$f=\frac{My}{I}=\frac{50,000\times22.2}{34,800}=31.9\ \frac{\text{lbs.}}{\text{ft.}}$$

The bending moment is negative, producing tension in the top and compression in the bottom of the envelope. Adding together the stresses due to longitudinal forces and bending, and dividing by cos α in order to make the correction for the longitudinal taper, it is found that:

At the top of the cross section, the longitudinal tension is

$$\frac{99.3+31.9}{.9962}=131.9\ \frac{\text{lbs.}}{\text{ft.}}=10.99 \text{ lbs./in.}$$

At the bottom of the cross section the longitudinal tension is

$$\frac{99.3-31.9}{.9962}=67.6 \text{ lbs./ft.}=5.63 \text{ lbs./in.}$$

TENSIONS IN CAR-SUSPENSION CABLES WHEN INCLINED LONGITUDINALLY.

It was shown in the preceding section that the airship designer is free within wide limits to assign arbitrarily the tensions in the car-suspension cables. Having chosen the tensions for any given condition, e. g., the airship at rest in horizontal trim with full load, the designer is not free to assign the change of tensions resulting from any change of load or trim or due to the application of another force to the car, such as propeller thrust.

The change in the cable tensions caused by any system of forces applied to the car may be calculated from the elastic changes in the lengths of the cable resulting from relative motion between the car and the hull, provided it is assumed that the points of attachment of the cables to the hull do not move relatively to each other, or move in a known manner. In rigid airships this assumption is justifiable. In non-rigids it is only approximately true owing to the flexibility of the envelope, and as the calculations are very laborious with the large number of suspension cables in such airships, it is generally better in practice to design the car suspension with an ample factor of safety in the normal condition, and with a good arrangement of martingales in accordance with approved practice. The martingales are suspension wires set at as flat an angle as practicable in order to take the longitudinal component of gravity when the airship is inclined longitudinally; and to be most effective there should be a pair leading forward from near the stern of the car, and another pair leading aft from near the bow.

The following method is suitable for an investigation into the changes in the tensions of the car-suspension cables of rigid airships, and may, if desired, be used as an approximation in non-rigid airships. Symmetry of the suspension in the horizontal direction usually makes it unnecessary to consider translation or rotation of the car with respect to the hull outside of the vertical plane, and in the following discussion such motions will not be considered, although the method may be extended to include them if required.

A small longitudinal movement of the car with respect to the hull produces in general a change not only in the longitudinal components of the suspension, but also in the perpendicular components and in the moments, so that the longitudinal motion will be accompanied by perpendicular motion and rotation. (Note: "Perpendicular" is used in this section instead of "vertical" to denote the direction in the vertical plane perpendicular to the longitudinal axis of the hull, in order to include cases when the airship is inclined longitudinally.) Similarly, rotation or perpendicular motion will in general be accompanied by the other two kinds of motion in the vertical plane. The problem is therefore to deduce three simultaneous equations which will give the net effect due to any change in the forces applied to the car.

Any change in the forces applied in the vertical plane may be resolved into three parts as follows:

Let A = Change in longitudinal force.
Let B = Change in perpendicular force.
Let M = Change in moment of the forces about a fixed point in the car.

The motion of the car with respect to the hull may be similarly resolved into three parts as follows:

Let x = Longitudinal motion.
Let y = Perpendicular motion.
Let ψ = Rotational motion.

The points of attachment of the suspension cables to the hull of the airship are assumed to be fixed relatively to each other.

The total change in the longitudinal components of the cable tensions due to a longitudinal movement of the car equal to some small unit of length (say 1 inch) must be calculated by adding together, with due regard to signs, the change in the longitudinal component of the tension in each cable.

From geometry and the laws of elasticity the change in the tension of a cable due to a longitudinal movement of 1 inch is given by:

$$T = \frac{Ea \cos \theta \cos \phi}{l}$$

Where T = Change of tension in pounds.

E = Modulus of elasticity of the cable in lbs./in.^{2}

a = Cross-sectional area of the cable in in.^{2}

l = Length of the cable in inches.

θ = Inclination of the cable to a plane through the longitudinal and transverse axes of the airship hull.

ϕ = Inclination of the cable to the verical plane.

By geometry the longitudinal component of T is $T \cos\theta \cos\phi$. The changes in the cable tensions and in the longitudinal and perpendicular components due to longitudinal and perpendicular movements of the car relatively to the hull are summarized in the following table:

Longitudinal component = Tension $\cos\theta \cos\phi$
Perpendicular component = Tension $\sin\theta \cos\phi$

For longitudinal movement x, tension $= \frac{Eax}{l} \cos\theta \cos\phi$

longitudinal component $= \frac{Eax}{l} \cos^2\theta \cos^2\phi$

perpendicular component $= \frac{Eax}{l} \sin\theta \cos\theta \cos^2\phi$

For perpendicular movement y, tension $= \frac{Eay}{l} \sin\theta \cos\phi$.

longitudinal component $= \frac{Eay}{l} \sin\theta \cos\theta \cos^2\phi$

perpendicular component $= \frac{Eay}{l} \sin^2\theta \cos^2\phi$

The change of tension in a suspension cable due to a small rotation in the vertical plane about some fixed point in the car, such as the C. G., is given by:

$$T = \frac{Eap}{l} \sin\psi \cos\phi$$

Where p is the distance from the C. G. to the line of action of the cable, measured in the vertical plane. The longitudinal and perpendicular components of the tension due to rotation are the same as in the case of tension produced by longitudinal or perpendicular motion.

The change in moment due to a change in the cable tensions caused by any of the three types of motion is equal to $Tp \cos\phi$.

Let a_1 = Sum of changes in longitudinal components of cable tensions due to a longitudinal movement of 1 inch.

Let b_1 = Sum of changes in perpendicular components of cable tensions due to a longitudinal movement of 1 inch.

Let m_1 = Sum of changes in moments of cable tensions due to a longitudinal movement of 1 inch.

Let a_2 = Sum of changes in longitudinal components of cable tensions due to a perpendicular movement of 1 inch.

Let b_2 = Sum of changes in perpendicular components of cable tensions due to a perpendicular movement of 1 inch.

Let m_2 = Sum of changes in moments of cable tensions due to perpendicular movement of 1 inch.

Let a_3 = Sum of changes in longitudinal components of cable tensions due to a rotation of 1° about the C. G.

Let b_3 = Sum of changes in perpendicular components of cable tensions due to a rotation of 1° about the C. G.

Let m_3 = Sum of changes in moments of cable tensions due to a rotation of 1° about the C. G.

All of the above quantities must be calculated, and the longitudinal, vertical, and rotational movements of the car with respect to the hull may then be obtained by solution of the following simultaneous equations of the first order which are derived directly from the definitions given above for the various forces, moments and motions:

$$\begin{aligned} a_1x+a_2y+a_3\psi &= -A \\ b_1x+b_2y+b_3\psi &= -B \\ m_1x+m_2y+m_3\psi &= -M \end{aligned}$$

After solving these equations for x, y and ψ, the total change of tension in each cable may be found by substituting the values of these variables in the preceding formulæ for cable tensions in terms of the variables and the known characteristics of each cable. To find the total tension in a cable, the change of tension must be added algebraically to the tension in the normal condition of the airship.

If the motion of the car relatively to the hull is sufficient to cause a cable previously taut to become slack, the effect of further motion must be calculated as if that cable were not present; i. e., the values of a_1, a_2, a_3, b_1, b_2, b_3, m_1, m_2, and m_3 must be modified by leaving out the cable which has become slack.

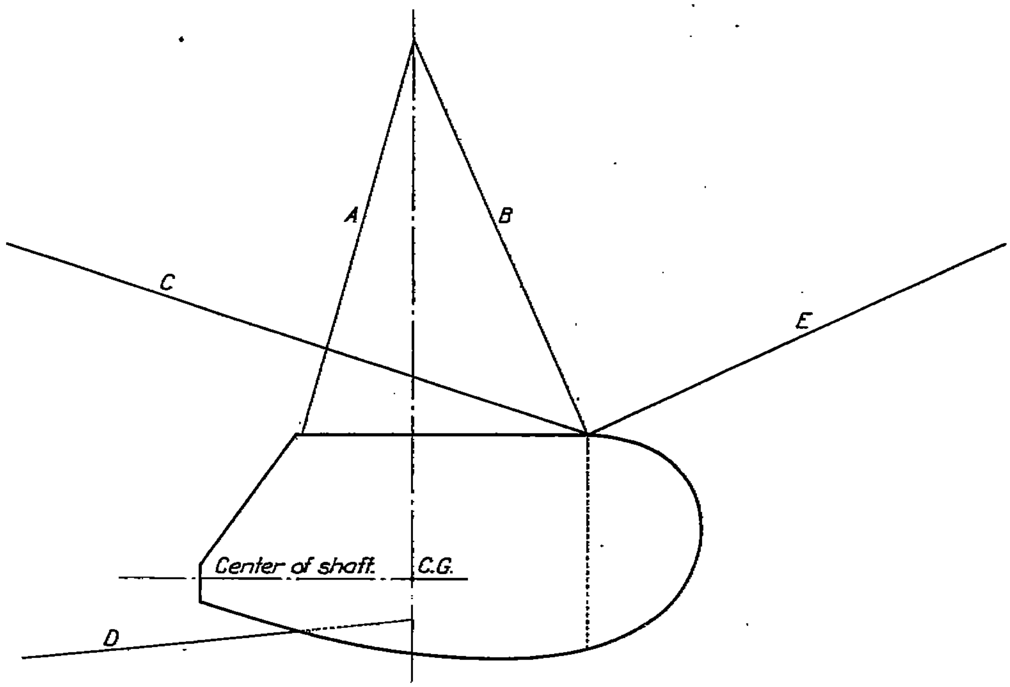


FIG. 3.

As a final check upon the accuracy of the work, force and moment diagrams should be constructed using the tensions found in the cables, and if the diagrams close it may be assumed that the work is correct.

Example: A side engine car of a rigid airship shown in the accompanying sketch (fig. 3) is suspended by five cables, designated A, B, C, D, and E. In the static condition with full load, the entire weight of the car, 2,800 pounds, is taken by the cables A and B, with the other three just barely taut. This arbitrary arrangement of the tensions in the cables is possible because A and B are secured to the hull of the airship at a point vertically over the C. G. of the car; but it should be noted that the division of the load between the cables is not determinate, and the designer is free to change the distribution of the load by assigning a tension to cables C and E, relieving B correspondingly.

Cables A, B, C, and E are all in the vertical plane. Cable D is inclined at an angle of 47° 45′ to that plane. In addition to the five cables there are three struts from the car meeting at a point on the main transverse frame of the airship abreast the C. G. of the car. These struts receive compression from the transverse component of any tension put upon the cable D, and they preserve the alignment of the car in the vertical plane, but they do not hinder small movements in that plane. The product Ea is assumed to be constant for all cables.

If a forward thrust is applied to the car, cables C and D are put in tension, and the tensions in cables A and B are modified. To calculate the effects of such a thrust a table is prepared of the known quantities in the three simultaneous equations required.

The convention with regard to signs is that forward and upward movements and anticlockwise rotation are counted positive, and the converse negative; and changes in components of cable tensions tending to produce positive motions are also counted positive, and conversely.

Characteristics of suspension cables of side power car of a rigid airship.

	Cable.				Sum.
	A	B	C	D	
ϕ	0	0	0	47° 45′	
$\cos \phi$	1.0	1.0	1.0	.672	
θ	74° 15′	65° 30′	18° 15′	−5° 45′	
$\sin \theta$	.9625	.9100	.3132	−.1002	
$\cos \theta$	.2715	−.4147	−.0497	−.9950	
l, 100 x in.	15.6	16.4	48.4	83.8	
p, 100 x in.	−.550	.835	.720	−.158	
T due to forward motion 1″ $\frac{Ea}{100,000}$ lbs.	−17.4	25.8	19.6	19.8	
Longitudinal components $\frac{Ea}{100,000}$ lbs.	−4.72	−10.49	−18.60	−13.22	−47.03 = a_1
Perpendicular components $\frac{Ea}{100,000}$ lbs.	−16.74	28.00	6.14	−1.33	11.07 = b_1
Moment $\frac{Ea}{100,000}$ in. lbs.	957	2110	1410	−210	4267 = m_1
T due to perpendicular rise of 1″ $\frac{Ea}{100,000}$ lbs.	−61.7	−55.5	−6.47	1.99	
Longitudinal components $\frac{Ea}{100,000}$ lbs.	−16.74	23.00	6.14	−1.33	11.07 = a_2
Perpendicular components $\frac{Ea}{100,000}$ lbs.	−59.4	−50.5	−2.02	−0.13	−112.05 = b_2
Moment $\frac{Ea}{100,000}$ in. lbs.	3390	−4635	−406	−21	−1732 = m_2
T due to rotation of 1° $\frac{Ea}{100,000}$ lbs.	61.7	−89.1	−26.0	5.5	
Longitudinal components $\frac{Ea}{100,000}$ lbs.	16.72	36.95	24.70	−3.68	74.69 = a_3
Perpendicular components $\frac{Ea}{100,000}$ lbs.	59.4	−81.0	−8.15	−0.37	−30.12 = b_3
Moment $\frac{Ea}{100,000}$ in. lbs.	−3390	−7445	−1870	−58	−12763 = m_3

Let it be assumed that the propeller exerts a forward thrust of 1,500 pounds, and since the line of action of this force passes horizontally through the C. G. of the car there is no change in the total moment about the C. G. or in the total perpendicular force. From the table, above, the coefficients of the variables are obtained, and the three simultaneous equations are written as follows:

$$-47.03x+11.07y+74.69\psi=-1{,}500\,\frac{100{,}000}{Ea}$$
$$11.07x-112.05y-30.12\psi=0$$
$$4{,}267x-1{,}732y-12{,}763\psi=0$$

Whence:

$$x=69\,\frac{100{,}000}{Ea}\text{ inches}$$

$$y=.63\,\frac{100{,}000}{Ea}\text{ inches}$$

$$\psi=23\,\frac{100{,}000}{Ea}\text{ degrees}$$

From the formulæ for the changes of tensions in the cables due to the three types of motions, the total change in the tension of each cable is given by:

$$T=Ea\,\frac{x}{l}\cos\theta\cos\phi+Ea\,\frac{y}{l}\sin\theta\cos\phi+Ea\,\frac{p}{l}\sin\psi\cos\phi=\frac{Ea}{l}\cos\phi\,(x\cos\theta+y\sin\theta+\psi p\sin 1^\circ)$$

By substituting the values of the quantities in this equation for each cable, the changes of tension are found to be as follows:

In cable A, $T=180$ pounds.
In cable B, $T=-342$ pounds.
In cable C, $T=750$ pounds.
In cable D, $T=1{,}470$ pounds.

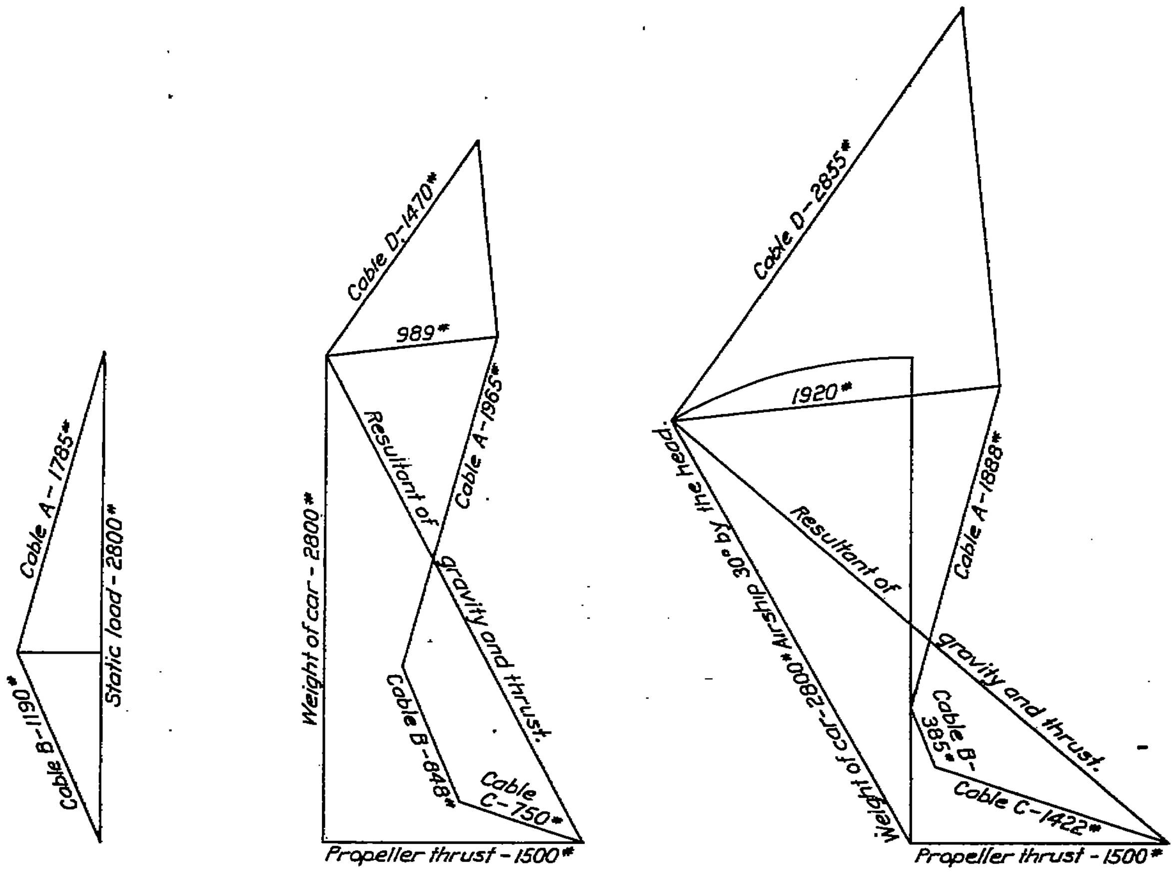


FIG. 4.

The tensions in cables A and B in the static condition are found from the force diagram in the accompanying figure to be 1,785 pounds and 1,190 pounds, respectively. The final tensions in the cables are:

In cable A, tension = 1,965 pounds.
In cable B, tension = 848 pounds.
In cable C, tension = 750 pounds.
In cable D, tension = 1,470 pounds.
In cable E, tension = 0 pounds.

If the airship noses down to an angle of 30° by the head, gravity exerts a longitudina force of $2{,}800\times \sin 30°=1{,}400$ pounds, and the perpendicular force of gravity is reduced by $2{,}800\ (1-\cos 30°)=375$ pounds. If in this condition the propeller is exerting a forward thrust of 1,500 pounds, the three simultaneous equations to determine the movement of the car with respect to the hull are:

$$-47.03x+11.07y+74.69\psi=-2{,}900\ \frac{100{,}000}{Ea}$$

$$11.07x-112.05y-30.12\psi=-375\ \frac{100{,}000}{Ea}$$

$$4{,}267x-1{,}732y-12{,}763\psi=0$$

Whence:

$$x = 131.5\frac{100{,}000}{Ea}\text{ inches}$$

$$y = 4.67\frac{100{,}000}{Ea}\text{ inches}$$

$$\psi = 43.4\frac{100{,}000}{Ea}\text{ degrees}$$

From these values of the variables the tensions in the cables may be calculated in the same manner as before, and the results are:

In cable A, tension = 1,888 pounds.
In cable B, tension = 385 pounds.
In cable C, tension = 1,422 pounds.
In cable D, tension = 2,855 pounds.
In cable E, tension = 0

Longitudinal forces to the rear due to reversing the propeller or inclination of the airship downward by the stern puts tension in the cable E, leaving cables C and D slack. The distribution of the load between cables A, B, and E is without redundancy of forces and may therefore be calculated by the conventional analytical or graphical methods for statically determinate problems.

CONCLUSION.

The theoretical calculations of the principal stresses in non-rigid airships due to the static forces now rest upon a basis nearly as well established as the parallel calculations for water-borne vessels. Much work remains to be done to determine the aerodynamic forces due to gusts of wind and the action of the rudders and elevators. Experimental apparatus is required to verify in actual flight the theoretical calculation of the stresses due to static forces, and more especially to determine the stresses due to dynamic forces which are very imperfectly susceptible of theoretical treatment.

Finally, there is need for a comprehensive literature to make a knowledge of the theory and practice of strength calculations for airships as accessible to the student as in the case of naval architecture.