REPORT No. 204

FORCES ON AIRSHIPS IN GUSTS

BY

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SUMMARY

The trials of the *Shenandoah* have proved that, as was previously suspected, the aerodynamic bending moments in approximately straight flight in gusty weather considerably exceed any which can be produced by maneuvers in still air. Hitherto it has been assumed that the conditions encountered in gusts could be approximately represented by considering the airship to be at an instantaneous angle of yaw or pitch (according to whether the gust is horizontal or vertical), the instantaneous angle being $\tan^{-1}(v/V)$, where v is the component of the velocity of the gust at right angles to the longitudinal axis of the ship, and V is the speed of the ship. In this report, prepared for publication by the National Advisory Committee for Aeronautics, it is shown that in determining the instantaneous angle of pitch or yaw the acceleration of the gust is as important as its maximum velocity. An expression is derived for this instantaneous angle in terms of the speed and certain aerodynamic characteristics of the airship, and of the maximum velocity and the acceleration of the gust, and the application of the expression to the determination of the forces on the ship is illustrated by numerical examples.

INTRODUCTION

No gust reaches its maximum velocity instantaneously. During the period of acceleration of the gust the airship is also acquiring an accelerated motion in the direction of the gust. If it is assumed that the rudders and elevators are so manipulated that there is no change in the direction of the longitudinal axis of the airship, the component v, of the instantaneous velocity of the gust at right angles to the longitudinal axis of the ship is attended by a transverse velocity u, in the ship, caused by the gust, and the instantaneous angle of yaw or pitch is given by $\alpha=\tan^{-1}\left(\frac{v-u}{V}\right)$. In practice an airship is rarely so well controlled that there is no change in the direction of its axis when running into a gust not in the direction of the longitudinal axis, and the extent to which she will yaw or pitch depends upon the skill of the helmsmen and can not be foretold; but it seems reasonable to consider the turning as superimposed upon the apparent angle of pitch or yaw to the gust when there is no angular motion of the longitudinal axis. In this report, only the latter condition is considered; the effect of steady turning has been dealt with in National Advisory Committee for Aeronautics Technical Notes Nos. 104, 105, 106 and 129; and angular acceleration opposed by the inertia of the hull has been considered in the paper on "The Strength of Rigid Airships," by Commander J. C. Hunsaker, and Messrs. C. P. Burgess and Starr Truscott, submitted for the *R-38* Memorial Prize Competition in 1923 and published in the Journal of the Royal Aeronautical Society June, 1924.

The velocity of the gust at right angles to the longitudinal axis of the airship at a time t after its commencement is assumed to be given by:

$$v=v_m(1-e^{-rt}) \tag{1}$$

where v_m is the maximum velocity, and r is a constant which determines the sharpness or acceleration of the gust. Gusts of this type are discussed by Prof. E. B. Wilson in a report entitled "Theory of an Aeroplane Encountering Gusts," constituting Part 2 of Report No. 1 of the National Advisory Committee for Aeronautics. He says: "If $r=1$, the gust has reached about two-thirds of its maximum value in one second; if $r=5$, the gust has reached two-thirds of its value in one-fifth of a second; if $r=1/5$, the two-thirds intensity is reached in 5 seconds. We may perhaps consider $r=1$ as giving a moderately sharp gust, $r=5$ as giving a very sharp, and $r=1/5$ as giving a tolerably mild gust."

CALCULATION OF THE INSTANTANEOUS ANGLE OF YAW OR PITCH

The velocity of the gust relatively to the airship in the direction at right angles to the longitudinal axis of the ship is $v-u$, and the accelerations of the gust and the ship produce a transverse force upon the bare hull equal to

$$F_1=\rho Q k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right)=\text{transverse force due to acceleration,}$$

where ρ = the density of the air,
Q = the volume of the hull,
k_2 = the coefficient of the additional mass of the hull in the transverse direction.

According to Munk's theory of aerodynamic forces at an instantaneous angle of pitch or yaw when the control surfaces are so manipulated as to prevent rotation of the airship, the transverse force upon the airship is equal to

$$F_2=\frac{\rho Q V^2 (k_2-k_1)\sin 2\alpha}{2a}=\text{transverse force due to angle of attack } \alpha,$$

where a is the distance from the center of buoyancy to the center of pressure on the tail surfaces, k_1 is the coefficient of the additional mass of the hull in the longitudinal direction, and the other symbols are as before. Let F be the total transverse force upon the airship, and by combining the forces due to the transverse acceleration and the angle of yaw or pitch,

$$F_1+F_2=F=\rho Q\left[\frac{V^2(k_2-k_1)\sin 2\alpha}{2a}+k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right)\right]. \tag{2}$$

Since F is opposed by the inertia of the ship against transverse acceleration,

$$F=\rho Q\,\frac{du}{dt}. \tag{2a}$$

Combining these two expressions, (2) and (2a)

$$\frac{du}{dt}=\frac{V^2(k_2-k_1)\sin 2\alpha}{2a}+k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right).$$

When α is small, $\sin 2\alpha=\tan 2\alpha=2\alpha$, approximately, and since $\tan\alpha=(v-u)/V$,

$$\frac{du}{dt}=\frac{V(v-u)\,(k_2-k_1)}{a}+k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right)=\frac{V(v-u)\,(k_2-k_1)}{a\,(1+k_2)}+\frac{k_2}{1+k_2}\cdot\frac{dv}{dt}.$$

Let

$$\frac{V(k_2-k_1)}{a\,(1+k_2)}=A \tag{3}$$

and since

$$v=v_m\,(1-e^{-rt})$$

$$\frac{dv}{dt}=v_m r e^{-rt}$$

$$\frac{du}{dt}=A\left\{v_m-u-\left[v_m-\frac{v_m k_2 r}{A\,(1+k_2)}\right]e^{-rt}\right\} \tag{4}$$

When

$$u=be^{-At}+ce^{-rt}+C$$

$$\frac{du}{dt}=-Abe^{-At}-rce^{-rt}.$$

Let

$$C=v_m,$$

and let

$$\frac{rc}{A}-c=v_m-\frac{v_m k_2 r}{A\,(1+k_2)}$$

$$=v_m\left(\frac{A+Ak_2-k_2r}{A+Ak_2}\right)$$

Then

$$\frac{du}{dt}=A\left[v_m-u-\left(\frac{A+Ak_2-k_2r}{A+Ak_2}\right)v_m e^{-rt}\right]$$

and

$$u=be^{-At}+v_m+\left[\frac{A+Ak_2-k_2r}{(r-A)\,(1+k_2)}\right]v_m e^{-rt}$$

when

$$t=O,\ u=O,\ \text{and}\ -b=\frac{v_m r}{(r-A)\,(1+k_2)}$$

whence

$$u=v_m\left[1-\frac{re^{-At}-(A+Ak_2-k_2r)\,e^{-rt}}{(r-A)\,(1+k_2)}\right] \tag{5}$$

Differentiating:

$$\frac{du}{dt}=\frac{v_m r}{(r-A)\,(1+k_2)}\,[Ae^{-At}-(A+Ak_2-k_2r)\ e^{-rt}] \tag{6}$$

By substituting in equation (4) the value of u given in (5), it is found that (4) and (6) are identical, proving that (5) is the correct value of u.

The instantaneous angle of yaw or pitch is a maximum when $v-u$ is a maximum, i. e., when

$$\frac{dv}{dt}-\frac{du}{dt}=O.$$

This occurs when

$$\frac{v_m}{(r-A)\,(1+k_2)}\,[Ae^{-At}-(A+Ak_2-k_2r)\ e^{-rt}]=v_m re^{-rt}$$

This expression reduces to

$$re^{-rt}=Ae^{-At} \tag{7}$$

Whence for maximum values of $v-u$ and α,

$$t=\frac{\log_e A-\log_e r}{A-r} \tag{8}$$

The value of A for any given problem is obtained from expression (3).

Combining expressions (1), (5) and (7), the maximum values of $v-u$ is given by:

$$(v-u)_{\max}=\frac{v_m re^{-rt}}{A\,(1+k_2)} \tag{9}$$

Example: Find the maximum instantaneous angle of yaw of the *Shenandoah* when flying at 88 ft./sec. and running into a gust having a maximum velocity of 20 ft./sec. normal to the longitudinal axis of the ship, and $r=1.0$.

For the *Shenandoah*, $k_2=0.950$, $k_1=0.026$, and $a=305$ feet. Whence

$$A=\frac{88\times 0.924}{305\times 1.95}=0.137$$

$$t=\frac{\log 0.137-\log 1.0}{0.137-1.0}=2.3 \text{ seconds}$$

$$v-u=\frac{20\times 1\times e^{-2.3}}{0.137\times 1.95}$$

$$=7.5 \text{ ft./sec.}$$

$$\tan\alpha=7.5/88=0.0852$$

$$\alpha=4°\ 52'$$

If the gust had been taken as attaining its maximum velocity instantaneously, $\tan\alpha$ would be $20/88=0.227$, and α would be 12° 48′ instead of only 4° 52′.

The foregoing study has dealt only with gusts varying with time, but it is probable that gusts or air currents varying with distance are more important. Vertical currents due to obstructions or convection are of this latter character. The variation of velocity in such gusts or air currents might be represented by an expression of the form:

$$v=v_m(1-e^{-rs})$$

where s is the distance over which the change of v is taken. Since the distance covered by the ship is Vt, the expression may be written as follows:

$$v=v_m(1-e^{-Vrt})$$

If s is assumed sufficiently large to produce a practically constant gust velocity along the entire length of the ship at any one moment, this case may be handled precisely as in the foregoing analysis, using the constant Vr instead of r.

Often the rate of change of the velocity of a gust or air current with distance is sufficiently rapid to produce a very considerable variation of the instantaneous angle of yaw or pitch along the ship; but since the bending moment is mainly due to aerodynamic forces concentrated upon a comparatively short length of the fore-body, it is believed to be reasonable to assume, even in this case, that the instantaneous angle is constant along the ship.

It is again emphasized that owing to the inability of the helmsmen to anticipate the magnitude and direction of gusts, an airship flying through gusty air is almost constantly subject to the combined effects of turning and instantaneous angle of yaw or pitch; and only the latter is considered in this report.

CALCULATIONS OF THE SHEARING FORCES AND BENDING MOMENTS IN AN AIRSHIP IN GUSTS

The transverse force upon an airship in a gust is shown by the preceding investigation to be made up of two parts. The part equal to $\rho Q k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right)$ is distributed along the hull according to the equation

$$\frac{dP_1}{dx}=\rho S k_2\left(\frac{dv}{dt}-\frac{du}{dt}\right)$$

where $\frac{dP_1}{dx}$ is the transverse air force per unit length along the hull, and S is the cross-sectional area of the hull. In other words, the air force due to the acceleration of the gust transversely to the hull is distributed in the same manner as the air displacement, and it is customary to assume in calculations of purely aerodynamic stresses that the weight and buoyancy are also distributed in this manner. Upon this assumption there are no shearing and bending forces from the acceleration of the gust, except in so far as the acceleration determines the instantaneous angle of pitch. Since the total force on the hull due to the acceleration must be small in proportion to the gross buoyancy, the shear and bending from this cause when the weight and buoyancy are unequally distributed are but small fractions of the static shear and bending.

The distribution of the transverse air force along the hull due to the instantaneous angle of yaw or pitch is given by Munk's formula, as follows:

$$\frac{dP_2}{dx}=\frac{\rho}{2}V^2(k_2-k_1)\sin 2\alpha\frac{dS}{dx}$$

The integration of this force gives a turning moment equal to $\frac{\rho Q}{2}V^2\ (k_2-k_1)\sin 2\alpha$, but no resultant force upon the bare hull. This turning moment divided by the distance from the center of buoyancy to the center of pressure on the tail surfaces gives the force on the tail surfaces and on the hull near these surfaces necessary to prevent angular motion of the ship. The point of application of the force is assumed to be the center of area of the tail surfaces. This force and the turning moment on the bare hull are balanced by transverse inertia forces distributed along the hull in the same manner as the weight and buoyancy which are assumed to be equal.

Example 2: Find the transverse forces producing shear and bending in the *Shenandoah* in the condition of example 1, assuming weight and buoyancy to be equally distributed.

Let the force assumed to be applied at the center of area of the tail surfaces be P, and by Munk's theory of forces at fixed angle of pitch or yaw it is given by

$$P=\frac{\rho Q}{2a}V^2\ (k_2-k_1)\sin 2\alpha$$

Let $\rho=0.00236$ slugs. For the *Shenandoah*, $Q=2{,}290{,}000$ ft.³: α was found in example 1 to be 4° 52′, whence $\sin 2\alpha=0.1687$, and the remaining quantities were given in example 1. Whence

$$P=\frac{0.00236\times 2{,}290{,}000\times 88^2\times 0.1687\times 0.924}{2\times 305}=10{,}700\text{ lb.}$$

The air force along the hull producing shear and bending is given by:

$$\frac{dP_2}{dx}=0.00118\times 88^2\times 0.1687\times 0.924\left(\frac{dS}{dx}\right)=1.43\frac{dS}{dx}$$

The sum of the transverse inertia forces along the hull equals the force P which is assumed to act through the center of area of the tail surfaces, provided these inertia forces do not include the accelerations of the additional mass of air and the weight of the ship which are equal and opposite and therefore offset each other. Since these forces are assumed to be distributed in the same manner as the volume, they are given by:

$$\frac{dP_3}{dx}=\frac{PS}{Q}=0.00466\ S$$

Summing up, the ship is subjected to shear and bending from three sets of transverse forces which are in equilibrium with each other as follows:

(*a*) A concentrated air force P, assumed concentrated at the center of area of the tail surfaces, equal to 10,700 pounds in this particular example.

(*b*) A distributed air force along the hull equal to 1.43 dS/dx.

(*c*) A distributed inertia force acting in the opposite direction to the air forces, and equal to 0.00466 S.

CONCLUSIONS

The forces upon an airship in a gust depend on the maximum velocity and acceleration of the gust, and the acceleration may be a function of either time or distance. The forces and stresses in the ship are also functions of the speed of the ship, varying nearly as the speed when the acceleration of the gust is a function of time, and nearly as the square of the speed when the acceleration is a function of distance.

Existing data on the structure of gusts is very meager, and a scientific investigation into the problem is very difficult and costly, and calls for a whole battery of electrically synchronized anemobiographs. Experiments with sensitive recording accelerometers on airships in service should afford valuable clews as to the magnitudes of the quantity du/dt from which the angle of pitch or yaw and the stresses in gusts could be inferred, and checked up with strain measurements.