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NOTES ON AERODYNAMIC FORCES ON AIRSHIP HULLS.

By L. B. Tuckerman,
Engineer Physicist, Bureau of Standards.

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Introduction.

For a first approximation the air flow around the airship hull is assumed to obey the laws of a perfect (i.e. free from viscosity) incompressible fluid. The flow is further assumed to be free from vortices (or rotational motion of the fluid).

These assumptions lead to very great simplifications of the formulae used but necessarily imply an imperfect picture of the actual conditions. The value of the results depends therefore upon the magnitude of the forces produced by the disturbances in the flow caused by viscosity with the consequent production of vortices in the fluid. If these are small in comparison with the forces due to the assumed irrotational perfect fluid flow the results will give a good picture of the actual conditions of an airship in flight.

* Dr. Max M. Munk's theory of the aerodynamic forces on an airship hull is presented in N.A.C.A. Technical Notes Nos. 104, 105 and 106. This paper was prepared by Dr. L. B. Tuckerman, a member of the special committee on the examination of the Naval Airship ZR-1, as a part of the Committee's report and as an interpretation and discussion of Dr. Munk's papers.

General.

The motion of a body through the fluid is accompanied with kinetic energy not only of its own motion but also of the motion of the fluid which it pushes aside. Since the fluid is assumed to be free from viscosity this kinetic energy of the fluid motion is not dissipated but accompanies the body in its motion, being transferred from portion to portion of the fluid as the body moves through it. The body, therefore, in any steady motion is accompanied by a steady configuration of fluid flow which changes only when the motion of the body changes. If the velocity of the body is increased in any proportion the velocity of all portions of the fluid is increased proportionately (provided the velocities are small in comparison with the velocity of sound in the fluid; this is true here since the fluid is assumed to be incompressible) and the kinetic energy of the accompanying fluid motion remains proportional to the kinetic energy of the body itself.

If, however, the character of the motion of the body changes, the shape of the accompanying fluid motion changes and the corresponding additional kinetic energy changes, although the velocity remain the same.

Pure Translation.

For a motion of pure translation Kirchhoff has shown that the kinetic energy (E_f) of the fluid can be written

$$2\,E_f = \rho\ K_x V_x^2 + \rho\ K_y V_y^2 + \rho\ K_z V_z^2 \qquad (1)$$

where x, y, and z are three special axes in the body, mutually perpendicular V_x, V_y and V_z the corresponding components of the velocity of the configuration and ρK_x, ρK_y and ρK_z are "added inertias" corresponding to these three directions. On K_x, K_y and K_z depend only the configuration of the body. The total kinetic energy E of the motion of the body is

$$2E = 2E_f + 2E_b = (\rho K_x+m)V_x^2 + (\rho K_y+m)V_y^2 + (\rho K_z+m)V_z^2 \qquad (2)$$

Since no energy is dissipated, any change in the total kinetic energy of the motion of the body must be due to work done on the body (or by the body)

$$-\delta W = \delta E = (\rho K_x+m)V_x\,\delta V_x + (\rho K_y+m)V_y\,\delta V_y + (\rho K_z+m)V_z\,\delta V_z \qquad (3)$$

If this change be due to a rotation of the body without change of total velocity

$$V_x^2 + V_y^2 + V_z^2 = V^2 = \text{constant}$$

and

$$V_x\,\delta V_x + V_y\,\delta V_y + V_z\,\delta V_z = 0$$

then $-\delta W = \delta E = (\rho K_x+\lambda)V_x\,\delta V_x + (\rho K_y+\lambda)V_y\,\delta V_y + (\rho K_z+\lambda)V_z\,\delta V_z$ (4)

where the Lagrangean multiplier λ may be given any value we please. In order that there be no moment acting on the body tending to produce this change it is necessary that $\delta E = T\,\delta\theta = 0$ where T = the moment of force acting on the body and $\delta\theta$ the angle of rotation. This equation can obviously be satisfied (provided $K_x \neq K_y \neq K_z \neq K_x$) in three and only three ways.

$$\lambda = -\rho K_x, \; V_y = V_z = 0$$
$$\lambda = -\rho K_y, \; V_z = V_x = 0 \qquad (5)$$
$$\lambda = -\rho K_z, \; V_x = V_y = 0$$

These three mutually perpendicular directions in the body are therefore directions of steady translation without the action of external moments.

Lateral transfer of momentum.

Consider a configuration of fluid flow A_1 (Fig. 1) having a resultant momentum M in the y direction and no resultant moment of momentum about the z-axis. Let this fluid motion be destroyed and replaced by an identical configuration in A_2 displaced a distance d having a component $d \sin \theta$ (where θ is the angle between the displacement and the direction of the resultant momentum) in the direction of the x-axis. To effect this change a negative resultant impulse $-M$ must be applied to the fluid in A_1 and a positive resultant impulse $+M$ to the fluid in A_2. That is, a resultant impulse moment $Md \sin \theta$ must act on the fluid. If, instead of a sudden transfer of momentum the transfer takes place continuously during the time t with a uniform velocity V such that $d = Vt$ the impulse moment $Md \sin \theta$ is due to a moment.

$$T = MV \sin \theta \qquad (6)$$

acting during the time t.

The distinction here between the momentum of the configuration of fluid flow and the momentum of a solid body should be noticed.

In a solid body the resultant momentum necessarily lies in the direction of its motion. The direction of resultant momentum of a configuration of fluid flow does not necessarily coincide with the direction of the motion of the configuration.

If $T = 0$ then $\theta = 0$ and the resultant momentum coincides in direction with the velocity.

In the three mutually perpendicular directions considered above, since there is no resultant moment of force, the resultant momentum of the fluid must coincide in direction with the velocity. In these three directions therefore, the momentum of the fluid is given by

$$M_x = \rho K_x V_x, \; M_y = \rho K_y V_y, \; M_z = \rho K_z V_z \qquad (7)$$

and the resultant momentum in any other uniform translation is the resultant of these three moments. In general, the resultant momentum M does not coincide in direction with the velocity of the body and thus needs a resultant moment $T = MV \sin \theta$ to be applied to the body in order to maintain a uniform motion of translation. This moment can be calculated either by

$$\frac{\partial E}{\partial \rho} = \frac{\partial E_f}{\partial \theta} = T \qquad (8)$$

(as in 4) or from $T = MV \sin \theta$ where $M \sin \theta$ is the transverse component of the momentum, (as in 6).

The calculation of the coefficients K_x, K_y and K_z for any given body solves therefore for that body the problem of the <u>total moments</u> necessary to maintain it in uniform translation at any angle of pitch and yaw.

If the motion of the body is confined to the xy plane and

$$K_y = K_2 \quad \text{and} \quad K_x = K_1, \quad \text{then}$$

$$2 E_f = \rho K_1 V_x^2 + \rho K_2 V_y^2 = \rho (K_1 \cos^2 \alpha + K_2 \sin^2 \alpha) V^2$$

where α is the angle of attack. Then

$$T = \frac{\partial E_f}{\partial \alpha} = 1/2 \; \rho V^2 \sin 2\alpha \, (K_2 - K_1) \qquad (9)$$

or, otherwise, from equation (7)

$$M_x = \rho K_1 V_x = \rho K_1 V \cos \alpha$$

$$M_y = \rho K_2 V_y = \rho K_2 V \sin \alpha$$

and the lateral component of the momentum

$$M \sin \alpha = M_y \cos \alpha - M_x \sin \alpha$$

$$= 1/2 \; \rho V \sin 2\alpha \, (K_2 - K_1)$$

and consequently, as before

$$T = VM \sin \alpha = 1/2 \; \rho V^2 \sin 2\alpha \, (K_2 - K_1) \qquad (9)$$

Force Distribution.

The determination of the force distribution which produces these moments requires a more detailed investigation.

General Method.

The general method may be sketched as follows:

Under the assumptions here made the fluid flow possesses a velocity potential φ such that the component velocities of the fluid (not of the configuration) at any point are given by:

$$v_x = -\frac{\partial \varphi}{\partial x},\ V_y = -\frac{\partial \varphi}{\partial y},\ V_z = -\frac{\partial \varphi}{\partial z}$$

having determined this velocity potential the pressure at each point of the surface is evaluated from the extended Bernouilli theorem

$$\frac{p}{\rho} = \frac{\partial \varphi}{\partial t} - 1/2\ v^2 - \Omega \qquad v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (10)$$

Here Ω is the potential of the external forces acting on the fluid. Since we are neglecting the change of pressure with height this may be treated as a constant. As Dr. Munk has shown, the term $\frac{\partial \varphi}{\partial t}$ may, if desired, be transformed into

$$\frac{\partial \varphi}{\partial t} = -\ V\ v\ \cos\ \theta \quad (11)$$

where V is the velocity of the configuration at the point and θ the angle between the velocity of the configuration and the velocity of the fluid.

This pressure is then integrated over the surface of successive zones of the ship, giving the resultant distribution of longitudinal and lateral forces along the ship.

This process although perfectly general in theory is generally impractical, since the velocity potential φ and consequently the velocity distribution has only been determined for a very few simple geometrical shapes, and even in these cases the computations are laborious.

Dr. Munk has, however, used the knowledge of the detailed pressure distribution based upon known velocity potentials in discussing the effect of changing shape upon flow around two dimensional shapes (No. 104, pp. 7, 8 and 9).

Approximate Solution.

To avoid these difficulties, Dr. Munk attacks the problem in the following approximate way: The flow about any portion of the elongated ship is considered to approximate at any given instant the corresponding flow about an infinite cylinder having the same cross-section. (Fig. 2). In this case the transverse added inertia is readily calculated from the well known case of two dimensional flow about an elliptic cylinder.

The velocity potential in this case is determined from the complex function

$$z = (x + i\ y) = f\ (w) = f\ (\varphi + i\ \psi)$$

where φ is the velocity potential and ψ the stream function. Here

$$z = A\ w \mp \frac{B}{w}.$$

Proper choice of the constants A and B fits this to any elliptic cylinder between the limits of the infinitely thin flat plate and the circle. (See Lamb's Hydrodynamics, 4th edition, p. 79, Lorenz, Technische Hydro-Mechanik, p. 287.)

If a and b are the major and minor semi-axes of the ellipse, the added inertia per unit length $\rho\ K_a' = \rho\ \pi\ b^2$ and $\rho\ K_b' = \rho\ \pi\ a^2$. In the special case of a circular cylinder to which he confines himself in this presentation

$$K_2' = K_a' = K_b' = D^2\ \frac{\pi}{4} = S \qquad (12)$$

where S is the cross-section of the ship at this point, K_1'

is of course zero. The contribution of any element of length dx to the total moment of the ship is therefore approximately from equation (9)

$$\frac{dT}{dx}\,dx = dT = 1/2\ \rho V^2 \sin 2\alpha\ (K_2' - K_1')dx = 1/2\,\rho V^2 \sin 2\alpha\, S\ dx \qquad (13)$$

since $\frac{dT}{dx}$ = shear and $\frac{d^2T}{dx^2}$ = lateral load per unit length, the total moment $T = 1/2\,\rho V^2 \sin 2\alpha \int S\ dx = 1/2\,\rho V^2 Q \sin 2\alpha$ (14)

where Q is the volume of the ship, and the lateral load $F = \int f\ dx$ is distributed according to the law

$$f\ dx = \frac{d^2T}{dx^2}dx = 1/2\ \rho V^2 \sin 2\alpha\ \frac{dS}{dx}\ dx \qquad (15)$$

This same method of reasoning he applies later to the problem of the rotating ship.

The same result is arrived at more directly as Dr. Munk explained verbally, as follows:

The transverse momentum of an element of length of the ship is, from equations (7) and (12) (Fig. 2)

$$\frac{dM}{dx}\,dx = dM = \rho\ V \sin\alpha\ S\ dx \qquad (16)$$

If the cross-section S were increasing at the rate $\frac{dS}{dt}$ the transverse momentum would be increasing at the rate

$$\frac{d(dM)}{dt} = \rho\ V \sin\alpha\ \frac{dS}{dt}\ dx = f\ dx$$

requiring a transverse load distribution f dx to impart this increase of momentum. The equivalent of this increase of cross-section is imparted to the transverse air flow by the longitudinal component of the ship's motion (Fig. 3). As shown in the diagram the air which was flowing about the section S is after a time dt flowing about the section $S + \frac{dS}{dt} dt$ where $\frac{dS}{dt} = V \cos \alpha \frac{dS}{dx}$.

The corresponding increase of transverse momentum must be imparted to it by a laterally distributed force on the ship.

$$f\,dx = \rho V \sin \alpha \, V \cos \alpha \frac{dS}{dx} dx$$

$$= 1/2 \, \rho V^2 \sin 2\alpha \frac{dS}{dx} dx \qquad (15)$$

as before.

The total moment on the ship calculated by this approximation was

$$T = 1/2 \, \rho V^2 Q \sin 2\alpha \qquad (14)$$

obviously here the volume replaces the coefficient $(K_2 - K_1)$ or equation (9).

These coefficients K_2 and K_1 have been calculated for a number of simple shapes. In particular, Lamb has calculated their value for ovary ellipsoids of different ratios of length to diameter.

In this case, for all finite lengths $K_2 - K_1$ is less than the volume. Dr. Munk therefore proposes to apply a correction factor $(k_2 - k_1)$ (where $k_2 = \frac{K_2}{Q}$ and $k_1 = \frac{K_1}{Q}$) to the preceding formula, thus giving

Total moment $T = 1/2 \, \rho V^2 (k_2 - k_1) \sin 2\alpha \; Q$ (17)

Shear $\frac{dT}{dx} = 1/2 \, \rho V^2 (k_2 - k_1) \sin 2\alpha \; S$ (18)

Lateral force $f\,dx = 1/2\ \rho\ V^2\ (k_2 - k_1)\sin 2\alpha \frac{dS}{dx}\,dx$ (19)

where k_2 and k_1 are Lamb's coefficients for the ellipsoid corresponding to the ship as calculated by the formula

$$\frac{L}{D}\ (\text{ellipsoid}) = \left(\sqrt{\frac{\pi}{6}\ \frac{L^3}{Q}}\right)(\text{ship}) \qquad (20)$$

ROTATION

General

If a body be in uniform translation parallel to one of its principal directions (V), (Fig. 4), the added momentum of the fluid will have the same direction. About any axis A' perpendicular to this direction there will be in general a resultant moment of momentum of the added momentum. There will, however, be a line BB' parallel to the direction of the velocity such that the resultant moment of momentum about any perpendicular axis (A) through it is zero. A similar line exists for translation in each of the other two "principal directions". These three lines do not in general intersect in a point. In bodies possessing certain types of aerodynamic symmetry, however, they intersect in a point C, the aerodynamic center of the body. If the body possesses geometrical symmetry this aerodynamic center lies on the planes or axes of symmetry. This aerodynamic center exists in airship hulls and will be used as the center of reference for points in the body. The axis of x will be laid through it in the "longitudinal" principal axis of the body, this axis being an axis of central symmetry.

The ship (Fig. 5) is supposed to be turning with a uniform angular velocity $\frac{V}{R}$ about a fixed azis O where V is the linear velocity of the aerodynamic center. The accompanying velocity configuration has a steady shape and steady speed and consequently a constant added energy but turns with the ship about the fixed center O. The constancy of the energy requires that the resultant of all the forces acting on the ship pass through the center O since otherwise the forces would have a moment about this axis and consequently add (or subtract) energy. These forces may be resolved into a radial (centripetal) air force F_r necessary to balance the centrifugal force of the ship and of the accompanying fluid and a tangential (inertial drag) force F_τ either positive or negative, which is added to the frictional drag (neglected here). The radial forces pass through O, but the tangential force F_τ considered as applied at the aerodynamic center requires an accompanying moment $F_\tau R$ to displace the line of action to O.

For the purpose of determining these forces the motion may be resolved into two parts, a parallel translation along the path and a rotation with angular velocity $\frac{V}{R}$ about the aerodynamic center. If the center of mass of the ship coincides with its aerodynamic center this latter motion will involve no resultant forces nor resultant moments and consequently the resultant forces are calculable from the parallel translation alone.

The total tangential momentum M_τ (Fig. 6) of the ship in parallel motion is composed of two parts, M_{τ_1} due to the mass m of the body

$$M_{\tau_1} = \rho V m \tag{21}$$

and M_{τ_2} due to the added tangential inertia

$$M_{\tau_2} = \rho V \, (K_2 \sin^2 \alpha + K_1 \cos^2 \alpha) \qquad (22)$$

while the total radial momentum M_r is the added radial momentum alone and is

$$M_r = 1/2 \, \rho V \, (K_2 - K_1) \sin 2\alpha \qquad (23)$$

then (see Fig. 6)

$$M_x = M_r \sin\theta + M_\tau \cos\theta \qquad M_\tau = M_{\tau_1} + M_{\tau_2}$$

$$M_y = M_r \cos\theta - M_\tau \sin\theta$$

From these the radial and tangential forces necessary to maintain the motion are

$$F_x = \frac{dM_x}{dt} = (M_r \cos\theta - M_\tau \sin\theta)\frac{d\theta}{dt} = \frac{V}{R}\,(M_r \cos\theta - M_\tau \sin\theta)$$

$$F_y = \frac{dM_y}{dt} = -(M_r \sin\theta + M_\tau \cos\theta)\frac{d\theta}{dt} = -\frac{V}{R}\,(M_r \sin\theta + M_\tau \cos\theta)$$

If $\theta = 0$ $F_x = F_\tau$ and $F_y = F_r$

Then $F_\tau = \frac{V}{R} M_r = 1/2 \, \rho V^2 \frac{1}{R} (K_2 - K_1) \sin 2\alpha$ (24)

This represents a drag when α is positive.

And $F_r = -\frac{V}{R} M_\tau = -\rho V^2 \frac{1}{R} m - \rho V^2 \frac{1}{R} (K_2 \sin^2\alpha + K_1 \cos^2\alpha)$ (25)

which is a centrifugal force.

This computation is of course exactly the same as the usual calculation of centrifugal force in rigid dynamics, the only difference being the existence of a transverse momentum, which gives rise to the "centrifugal" drag force. This is a generalized centrifugal force in the Lagrangean sense.

The drag F_T is wholly due to air forces acting on the ship but of the centrifugal force F_r that part due to the mass of the ship $\rho V^2 \frac{1}{R} m$ involves no air forces, the added centrifugal force $\rho V^2 \cdot \frac{1}{R} (K_2 \sin^2 \alpha + K_1 \cos^2 \alpha)$ however, is transmitted to the ship by air forces acting on it.

The drag F_T considered applied at the aerodynamic center is accompanied by the moment $F_T R = 1/2 \rho V^2 (K_2 - K_1) \sin 2\alpha$ which is the same as the unstable moment in rectilinear motion (equation (9)). The maintenance of the motion demands therefore (Fig. 7) a resultant force F and a moment T in addition to the aerodynamic forces here discussed. The fins alone supply the transverse component F' and the moment T = F'a.

Distribution of these forces.

Dr. Munk calculates the distribution of these air forces by the first method used in the case of rectilinear motion. Here, however, it is necessary to bear in mind that because of the curvature of the path the effective angle of attack of successive elements of the ship's length are different.

These angles of attack may be calculated as follows (See Fig. 8).

$$\frac{x}{\sin \theta} = \frac{R}{\sin(\alpha' + \frac{\pi}{2})} \doteq \frac{R}{\cos \alpha'}$$

$$\alpha' = \alpha - \theta = \alpha - \text{arc sin} \left(\frac{x}{R} \cos \alpha' \right)$$

Then $\sin 2\alpha' = \sin 2\alpha \cos 2 \left(\text{arc sin} \frac{x}{R} \cos \alpha'\right)$

$$- \cos 2\alpha \sin 2 \left(\text{arc sin} \frac{x}{R} \cos \alpha'\right)$$

If α and $\frac{x}{R}$ are both small this reduces to

$$\sin 2\alpha' = \sin 2\alpha - 2\frac{x}{R} \tag{26}$$

Then each element of length dx contributes an element of moment

$$\frac{dM}{dx} dx = 1/2 \rho V^2 (\sin 2\alpha - 2\frac{x}{R}) S dx \tag{27}$$

The first term is due to the translation alone and the second term to the added rotation combined with the translation. Dr. Munk calculates these terms separately but the reasoning is equivalent to that here given. The total amount is

$$T = 1/2 \rho V^2 \sin 2\alpha \int S dx - \rho V^2 \frac{1}{R} \int S x dx \tag{28}$$

The first term is the unstable moment of the translational motion, and the second term is zero since $\int S x dx$ is the static moment of the volume about the aerodynamic center, which on the assumptions here made coincides with the center of volume. As before, this calculation gives a resultant moment somewhat larger than acts on a ship of finite length so that he introduces again the correction factor $(k_2 - k_1)$ in the first term.

This factor gives the correct resultant moment. Since the remaining terms have no resultant, nor resultant moment, there is no obvious correction factor. Dr. Munk uses here k_2* as a correction factor instead of $(k_2 - k_1)$.

The force distribution is then

*Note: The difference is not great and it is all a matter of judgment but Dr. Munk's reason for using a different correction factor here is not clear to me. The forces are all calculated on the same basis of approximation. L.B.T.

$$\frac{d^2M}{dx^2}dx = f\ dx = 1/2\,\rho V^2(k_2-k_1)\sin 2\alpha \frac{dS}{dx}dx - \rho V^2\frac{k_2}{R}(x\frac{dS}{dx} + S)dx \qquad (29)$$

and the total transverse force

$$F = \int f\ dx = 0$$

This approximate distribution of transverse air forces therefore accounts for the resultant unstable moment of the ship. It of course does not account for the drag. The undermined drag forces are, however, small, and being longitudinal, give rise to no appreciable bending moments in the hull.

In addition, however, the approximation has yet to account for the added centrifugal force (equation (25)).

$$\rho V^2 \frac{1}{R} (K_2 \sin^2\alpha + K_1 \cos^2\alpha)$$

This force is of course small since α and K_1 are both small. For an $\frac{L}{D}$ ratio of 6 and an angle of attack of 8 degrees it is less than 6 per cent of the ship's own centrifugal force.

Of the two parts of this added centrifugal force, the first

$$\rho V^2 \frac{K_2}{R} \sin^2\alpha = \rho V^2 \frac{k_2}{R} Q \sin^2\alpha$$

being due to the transverse added inertia can reasonably be assumed to be distributed according to the cross sectional area or

$$f\ dx = \rho V^2 \frac{k_2}{R} S \sin^2\alpha \qquad (30)$$

The second term $\rho V^2 \frac{K_1}{R} \cos^2\alpha = \rho V^2 \frac{k_1}{R} Q$ (31)

(α being small $\cos^2\alpha = 1$ approx.)

requires a more detailed treatment, since its longitudinal distribution might give rise to considerable bending moments. As this term arises from the longitudinal added inertia alone, he considers a case of longitudinal flow only, the flow arising from a single source and equal sink (Fig. 9). He chooses this flow (which gives a blunter airship model) instead of the corresponding ellipsoid because of the simpler mathematical treatment. The corresponding velocity is

$\varphi = \frac{VD^2}{16}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ (see Fig. 9) with the velocity distribution

$$v_x = -\frac{\partial\varphi}{\partial x} \frac{VD^2}{16}\left(\frac{x - c}{r_1^3} - \frac{x + c}{r_2^3}\right)$$

Here $L = 2c + \frac{D}{\sqrt{2}}$ approximately

or nearly $L = 2c$.

As may be seen from the indicated line of flow the longitudinal component of the velocity and consequently the added inertia is positive near the two ends but negative along nearly the whole of the side of the ship. At mid-section this negative velocity is approximately $\frac{VD^2}{2L^2}$ diminishing to about $\frac{VD^2}{16L^2}$ opposite the two sources and then rapidly changing sign around the nose. To simplify his computation Dr. Munk assumes that it maintains its mid-section value $\frac{VD^2}{2L^2}$ along the whole length and that the transverse velocity is negligible. This obviously results in an over-estimation of the bending moments produced. This flow, however, represents a pure translation. The ship actually is rotating about a center O (Fig. 10), so that if V is the ship's velocity at the aerodynamic center, the surface velocity of the ship changes across the ship having a velocity $V' = (V + y\frac{V}{R})$ at any point a horizontal distance y from the center. Dr. Munk* assumes that the air velocity remains the same in the circular flight as in straight flight,** which gives

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* In a personal conversation, Dr. Munk states that this method of reasoning is different from the one he used, but as it arrives at the same result, is presumably equivalent to it.

**Note: If the alternative assumption be made that the air velocity at any point of the surface in circular flight bears the same ratio to the surface velocity of the ship as it does in straight flight then

$$v' = -\frac{VD^2}{2L^2}\left(1 + \frac{y}{R}\right)$$

$$V' = V\left(1 + \frac{y}{R}\right)$$

and

$$p = \frac{\rho V^2 D^2}{2\,L^2}\left(1 - \frac{D^2}{4L^2}\right)\left(1 + \frac{y}{R}\right)^2$$

and the pressure gradient

$$\frac{dp}{dy} = \frac{\rho V^2 D^2}{R\,L^2}\left(1 - \frac{D^2}{4L^2}\right)\left(1 + \frac{y}{R}\right) = \frac{\rho\,V^2 D^2}{R\,L^2} \text{ approximately.}$$

This pressure gradient is twice as great as on Dr. Munk's assumption. It seems probable that the actual air velocity will lie between these two extremes, so that Dr. Munk's assumption represents an under-estimation of the pressure gradient and consequently an under-estimation of the bending moments. As noted above, the assumption that the air velocity maintained its mid-section velocity $\frac{VD^2}{2L^2}$ along the whole length, caused an over-estimation of the bending moments. These two factors will of course partially compensate each other, so that the Munk's assumption is probably more nearly correct.

air velocity $v' = v = -\frac{VD^2}{2L^2}$

configuration velocity $V' = V\ (1 + \frac{Y}{R})$.

Since the transverse air velocity is considered negligible

$\theta = 0$ and $\cos\hat{\theta} = 1$, then the pressure (equation (10) combined with (11))

$$p = -\frac{\rho}{2} v'^2 - \rho V' v' \cos\theta + \text{constant}$$

gives

$$p = -\frac{\rho}{2}\left(\frac{VD^2}{2L^2}\right)^2 + \rho\,[V\ (1 + \frac{Y}{R})]\quad[\frac{VD^2}{2L^2}] + \text{constant}$$

and the pressure gradient

$$\frac{dp}{dy} = \frac{\rho V^2 D^2}{2\,R\,L^2}$$

This pressure gradient acts in the same way as a gravitational pressure gradient due to a fluid of density $\frac{\rho V^2 D^2}{2\,R\,L^2}$ in a field of horizontal intensity 1. The total lateral force is then $\frac{\rho V^2 D^2 Q}{2\,R\,L^2}$ (32) and is distributed along the ship proportional to the cross-sectional area. The added centrifugal force $\frac{\rho V^2}{R} Qk_1$ consists therefore of two centrifugal forces $\frac{1}{2}\frac{\rho V^2}{R} Q\ (k_1 + \frac{D^2}{2L^2})$ (33) concentrated practically at the ends of the ship combined with a centripetal force $\frac{\rho V^2}{R} Q \frac{D^2}{2L^2}$ (32) distributed along the ship proportional to the cross-sectional area. The factor $\frac{\rho V^2}{R} Q$ is of course the centrifugal force of the ship itself, when in a state of static equilibrium. For $\frac{L}{D} = 6$, $k_1 = .045$ and $\frac{D^2}{2L^2} = .014$.

Transverse force on the fins.

A small part of the centrifugal force can be balanced by the lateral viscous drag of the ship but the larger portion must be balanced by the lateral force on the fins. In addition, this lateral force must neutralize the unstable moment of the ship (Fig, 7). In his computation Dr. Munk assumes this lateral force equal to the centrifugal force of the ship alone. This either neglects the added centrifugal force or considers it neutralized by the lateral viscous drag. Equating moments (see Fig. 7)

$$\rho V^2 Q \frac{a}{R} = \rho V^2 Q \, 1/2 \, (k_2 - k_1) \sin 2\alpha$$

or

$$(k_2 - k_1) \sin 2\alpha = \frac{2a}{R} \qquad (34)$$

Summary.

The lateral forces acting on the ship are then:

1. The forces producing the unstable moment due to angle of attack

$$T = 1/2 \, \rho V^2 (k_2 - k_1) \sin 2\alpha \quad Q \qquad (17)$$

$$= \frac{\rho V^2}{R} a \, Q \qquad (35)$$

The forces producing this moment are distributed according to the law

$$f \, dx = \frac{\rho V^2}{R} a \frac{dS}{dx} dx \qquad (36)$$

2. The lateral forces due to rotation combined with tangential velocity. These forces have no resultant and no resultant moment. They are distributed according to the law

$$f\,dx = -\frac{\rho V^2}{R} k_2 \left(x \frac{dS}{dx} + S\right) dx \qquad (29)$$

3. The centrifugal forces on the ship itself $\frac{\rho V^2}{R} Q$ (25)

provided the ship is in static equilibrium. If in addition the mass of the ship is distributed longitudinally proportional to the cross section these are distributed according to the law

$$f\,dx = \frac{\rho V^2}{R} S\,dx \qquad (37)$$

These nearly neutralize the second term of (2)*.

4. The added centrifugal force due to the added longitudinal inertia

$$\frac{\rho V^2}{R} k_1 Q \qquad (31)$$

This is distributed approximately as a concentrated load

$$\frac{\rho V^2}{R} \frac{Q}{2} \left(k_1 + \frac{D^2}{2L^2}\right) \qquad (33)$$

at each end and a load distributed according to the law

$$f\,dx = -\frac{\rho V^2}{R} \frac{D^2}{2L^2} S\,dx \qquad (38)$$

5. The added centrifugal force due to the added transverse inertia

$$\frac{\rho V^2}{R} k_2 Q \sin^2 \alpha \qquad (25)$$

This is distributed according to the law

$$f\,dx = \frac{\rho V^2}{R} k_2 \sin^2 \alpha\, S\,dx = \frac{\rho V^2}{R} \frac{a^2}{R^2} \frac{k_2}{(k_2 - k_1)^2} S\,dx \qquad (30)$$

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*Note: For any other mass distribution it would be of course easy to calculate the corresponding force distribution. Since normally the static bending moments of the hull are everywhere hogging moments, the actual force distribution is somewhat greater at the ends and less in the middle. L.B.T.

6. The lateral force on the fins practically concentrated at the center of pressure of the fins

$$-\frac{\rho V^2}{R} Q \qquad (25)$$

The sum total of all forces is then:

Three concentrated loads

a) at front end outward $\frac{\rho V^2}{R} \frac{Q}{2} \left(k_1 + \frac{D^2}{2L^2}\right)$ (33)

b) at center of pressure of fins inward $\frac{\rho V^2}{R} Q$ (25)

c) at rear end outward $\frac{\rho V^2}{R} \frac{Q}{2} \left(k_1 + \frac{D^2}{2L^2}\right)$ (33)

And a force distributed along the ship, with the resultant outward intensity

$$f = \frac{\rho V^2}{R} [(a - k_2 x) \frac{dS}{dx} + (1 - k_2 - \frac{D^2}{2L^2} + k_2 \sin^2 \alpha) S]$$

$$= \frac{\rho V^2}{R} [(a - k_2 x) \frac{dS}{dx} + \left(1 - k_2 - \frac{D^2}{2L^2}\right) S + \frac{a^2}{R^2} \frac{k_2}{(k_2 - k_1)^2} S] \qquad (39)$$

<u>Note</u>:

The method of reasoning used in these papers introduces discrepancies between the computed forces and the actual forces due to two things:

1) The viscosity of the air is assumed to be zero with the consequent elimination of all viscous drag.

These discrepancies in the present state of the theory can probably only be estimated by comparison with experiment.

2) The transverse flow about any element of the ship is assumed to be the same as that about the corresponding portion of an infinite cylinder. This assumption is most accurate when $\frac{1}{D} \frac{dS}{dx}$

is small. It will represent most closely the conditions amidships $\left(\frac{1}{D} \frac{dS}{dx} = 0\right)$. The largest discrepancies will occur near the blunt nose of the ship $\left(\frac{1}{D} \frac{dS}{dx} = \infty\right)$ and the next largest near the tail, where $\frac{1}{D} \frac{dS}{dx}$ is finite but large.

Since even small discrepancies in forces near the ends may result in relatively large discrepancies in the bending moments on the ship, it would seem to be very desirable to have some comparison of the results of this approximate method with an accurate computation of the forces on a shape approximating that of the airship.

The theory of the potential flow about an ovary ellipsoid is so complete that it is possible (although tedious) to compute the actual force distribution along such a shape both for straight flight and steady turning.

It would seem that the comparison of the results of such a computation with the results of the approximate analysis given above would be of value in indicating the magnitude of the discrepancies involved.

L. B. Tuckerman.

Supplementary Note No. 1. Modification of Dr. Munk's formulae.

Mr. C. P. Burgess has called my attention to the practical disadvantage of an approximate load distribution which is not in equilibrium. By neglecting the added centrifugal forces in the calculation of the lateral force on the fins Dr. Munk leaves an unbalanced outward force of $\frac{\rho V^2}{R} Q (k_1 + k_2 \sin^2 \alpha)$. This makes no appreciable difference in the resulting moments on the ship but is inconvenient in practical computation, since it prevents the check obtained by computing both ways along the hull.

This may be avoided by using the total centrifugal force in calculating the fin load, i.e.

$$\frac{\rho V^2}{R} Qa(1 + k_1 + k_2 \sin^2 \alpha) = \rho V^2 Q \; 1/2 \; (k_2 - k_1) \sin 2\alpha$$

or

$$(k_2 - k_1) \sin 2\alpha = \frac{2a}{R} (1 + k_1 + k_2 \sin^2 \alpha)$$

Since α is small the second approximation of its value will be sufficiently close for a numerical check. Then the total forces on the ship become:

a) at bow outward $\qquad \frac{\rho V^2}{R} Q \; 1/2 (k_1 + \frac{D^2}{2L^2})$

b) at center of pressure of fins inward $\qquad \frac{\rho V^2}{R} Q (1 + k_1 + k_2 \sin^2 \alpha)$

c) at stern outward $\qquad \frac{\rho V^2}{R} Q \cdot 1/2 (k_1 + \frac{D^2}{2L^2})$

and a force distributed along the ship with the resultant outward intensity:

$$f = \frac{\rho V^2}{R} [\{a(1 + k_1 + k_2 \sin^2 \alpha) - k_2 x\} \frac{dS}{dx} + (1 - k_2 + k_2 \sin^2 \alpha - \frac{D^2}{2L^2}) S]$$

Supplementary Note No. 2. Discrepancy between Dr. Munk's Theory and N.P.L. Estimates.

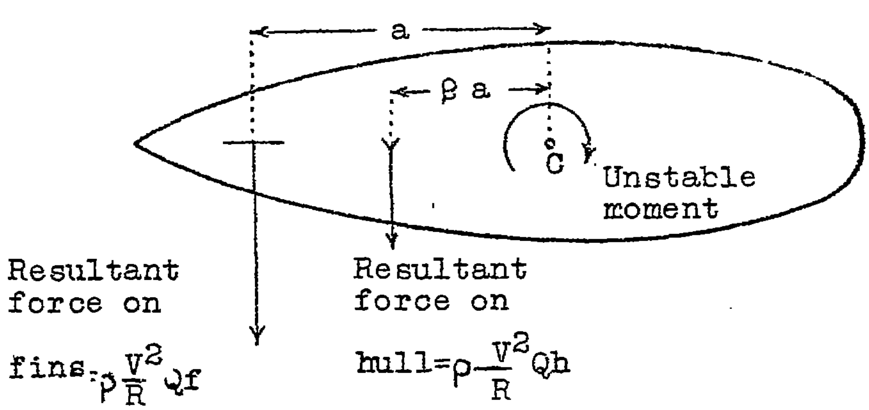
In computations for the ZR-1, Mr. Burgess has noted some discrepancy between Dr. Munk's theory and N.P.L. Estimates based on model tests. He pointed out that it is at least partially explained by the neglect in Dr. Munk's theory of the lateral resultant force on the hull arising from viscosity. The N.P.L. results show

$$h = \frac{\text{Force on hull}}{\text{Total force}} = \frac{3400}{9600} = .354$$

$$f = \frac{\text{Force on fins}}{\text{Total force}} = \frac{6200}{9600} = .646$$

The lateral force on the hull is thus over 1/3 the total force and would make a considerable change in the results.

It seems that the following method might give a somewhat better approximation. Assume forces as indicated in the diagram.



Then equating moments:

$$\frac{\rho V^2}{R} Q a (f + \beta h) = \rho V^2 Q \, 1/2(k_2 - k_1) \sin 2 \alpha_1$$

or

$$(k_2 - k_1)\sin 2\alpha_1 = \frac{2a}{R}(f + \beta h)$$

where Dr. Munk found

$$(k_2 - k_1)\sin 2\alpha_c = \frac{2a}{R}$$

then

$$\frac{\sin 2\alpha_1}{\sin 2\alpha_0} = f + \beta h$$

or

$$\beta = \frac{\frac{\sin 2\alpha_1}{\sin 2\alpha_0} - f}{h}$$

Burgess gives $\alpha_1 = 7^\circ\ 12'$; $\alpha_0 = 8^\circ\ 45'$

Substituting values

$$\beta = \frac{\frac{.2487}{.3007} - .646}{.354} = 0.51$$

The lateral forces on the hull have then apparently a resultant applied about half way between the center of buoyancy and the center of pressure of the fin.

It would seem then that a recomputation by Dr. Munk's method based on an angle of yaw of 7° 12' with the addition of some reasonable distribution of lateral forces on the hull with a resultant at 0.51a might give a still closer approximation to the actual forces in a steady turn.

Supplementary Note No. 3. Approximate Formulae for Lamb's Coefficients.

In comparing airships of different fineness ratio the variation of Lamb's coefficients k_1, k_2 and $k_1 - k_2$ may not always be negligible, although this variation need not be accurately estimated. For such cases it may be worth while noting the linear approximations given on the accompanying figures. These cover the whole range with a maximum error of 5% of the volume or the range $4 < \frac{L}{D} < \infty$ with a maximum error of 2%.

It is of course obvious that in the range $4 < \frac{L}{D} < \infty$ parabolic approximations would give still closer values. For instance, in this range the approximation $k_2 - k_1 = 1 - 1.53 \left(\frac{D}{L}\right)^{1.4}$ has a maximum error of less than 0.3%. In view of the roughness of the other approximations involved the accuracy gained is probably not worth the extra labor.

L.B.T.

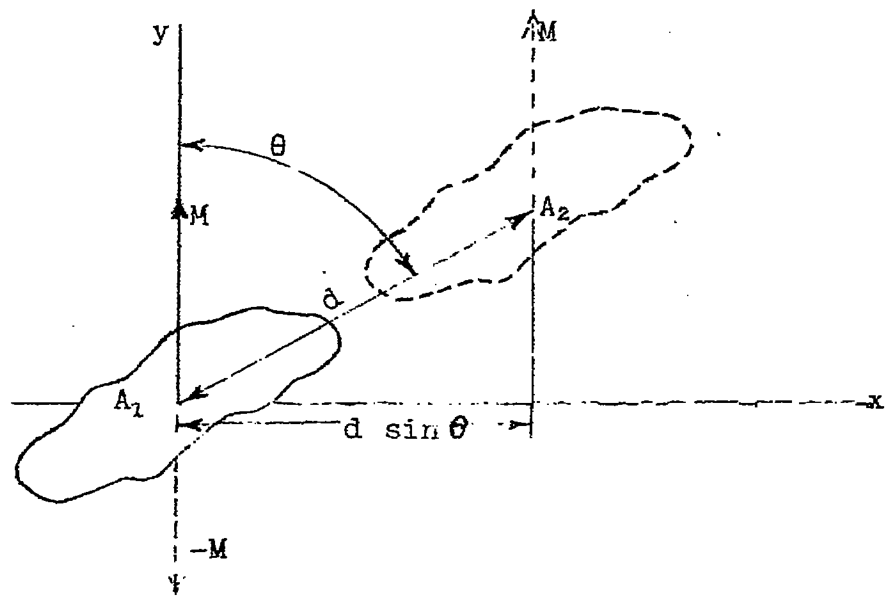


Fig.1

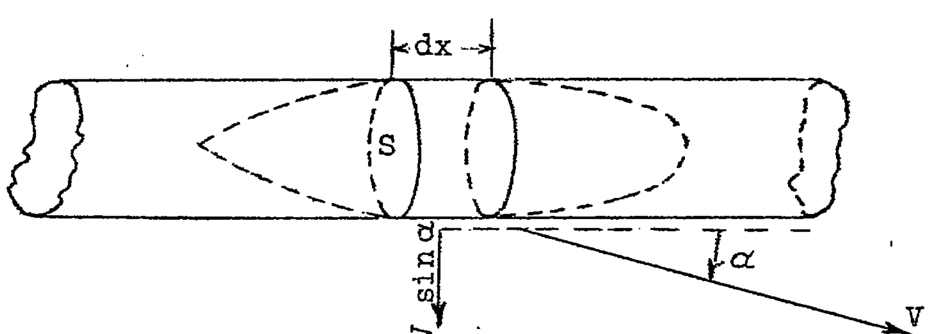


Fig.2

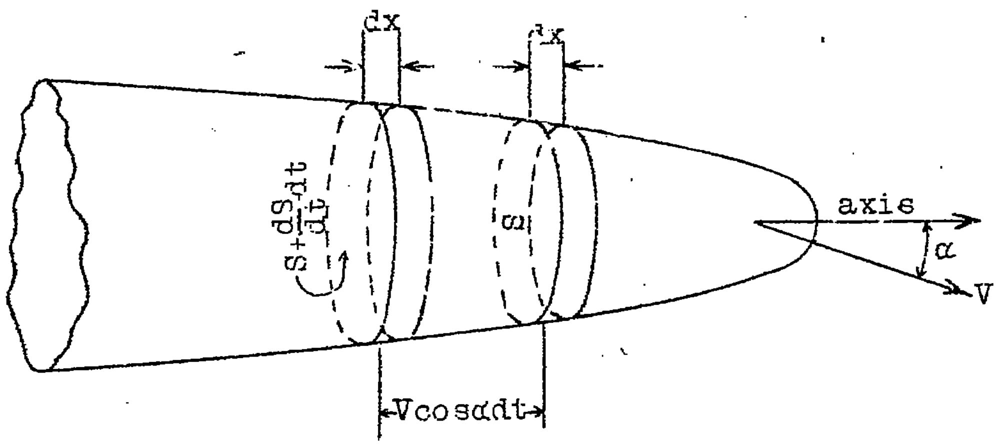


Fig.3

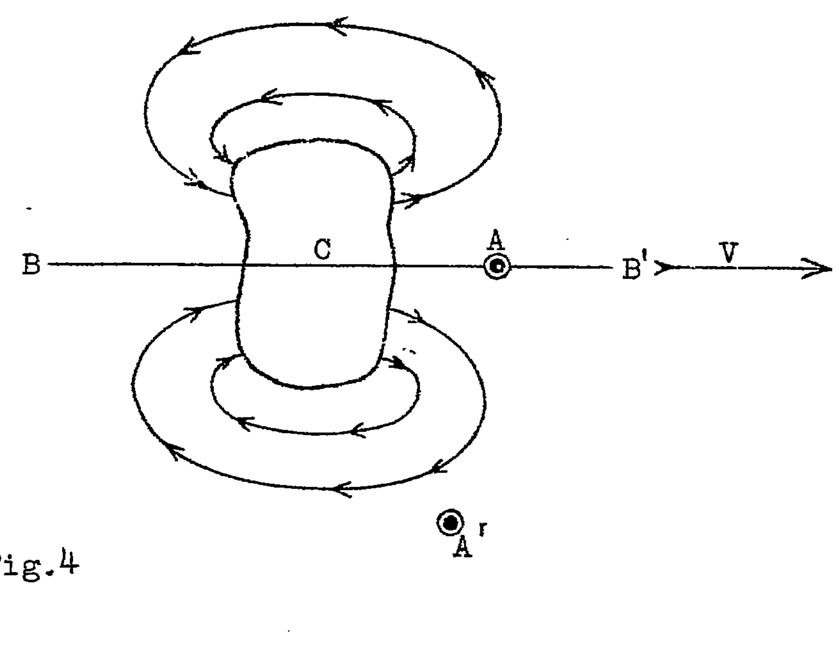


Fig.4

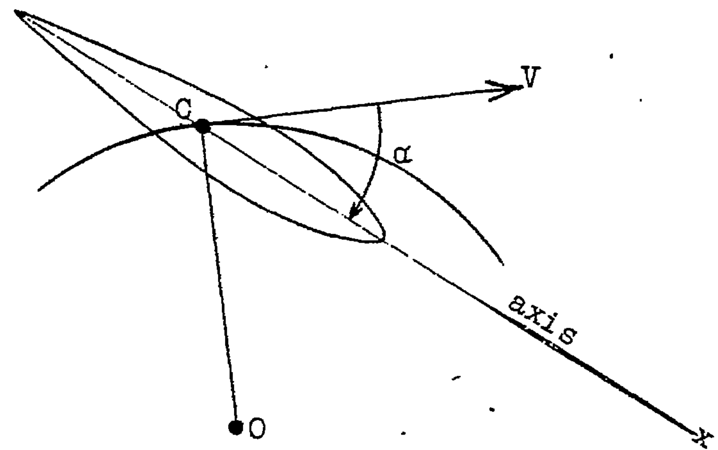


Fig.5

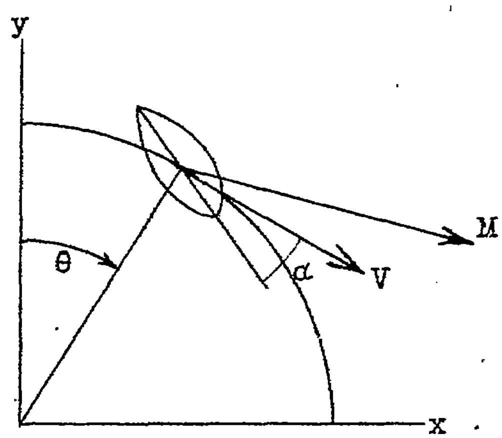


Fig.6

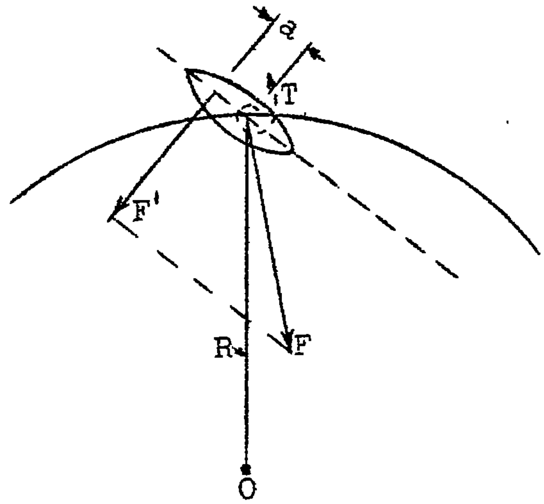


Fig.7

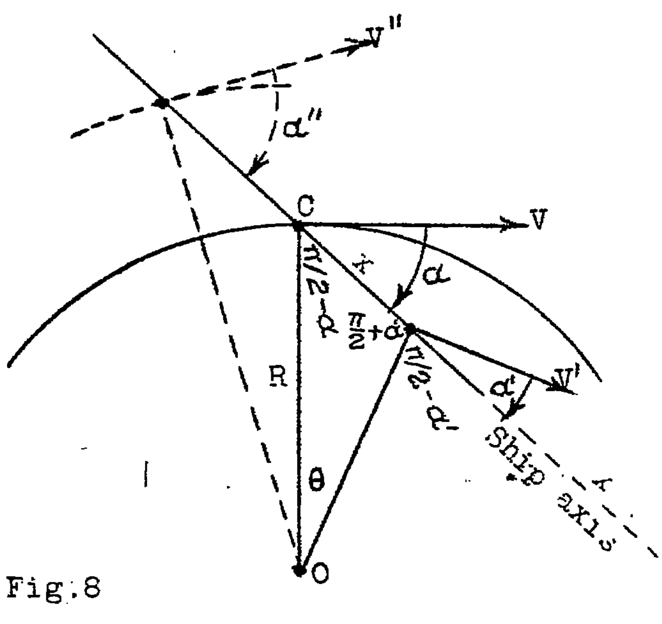


Fig.8

Figs.7 and 8

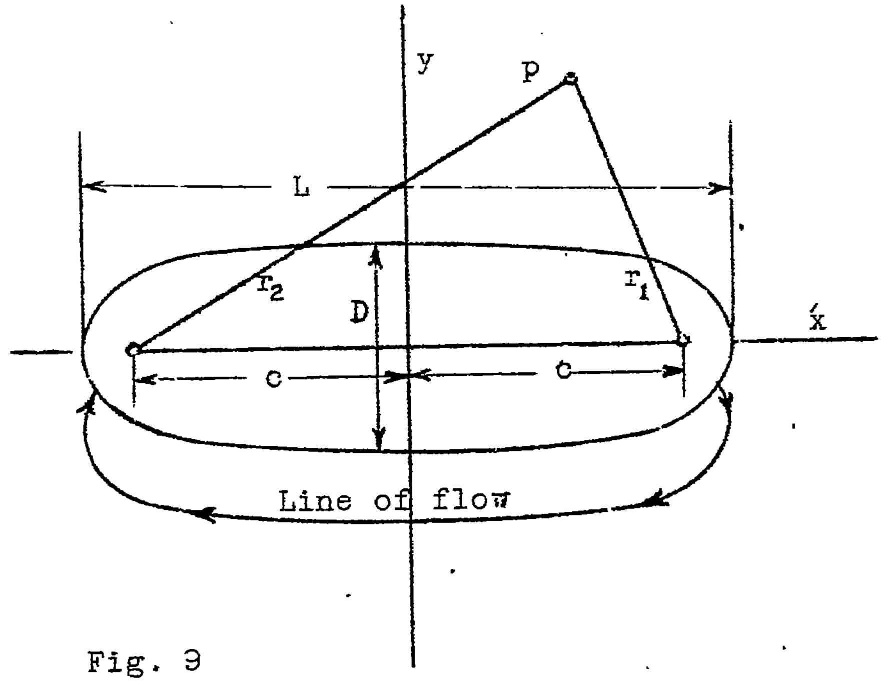


Fig. 9

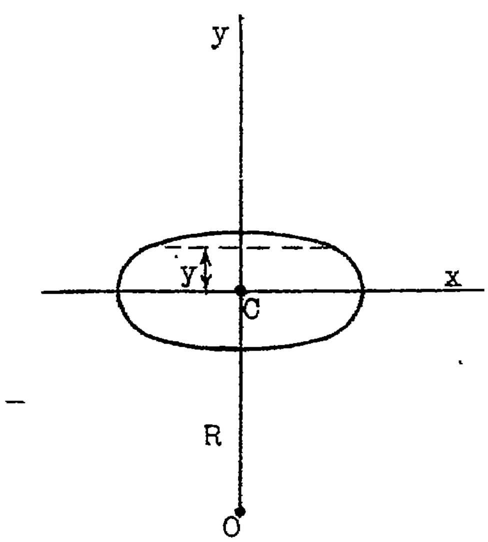
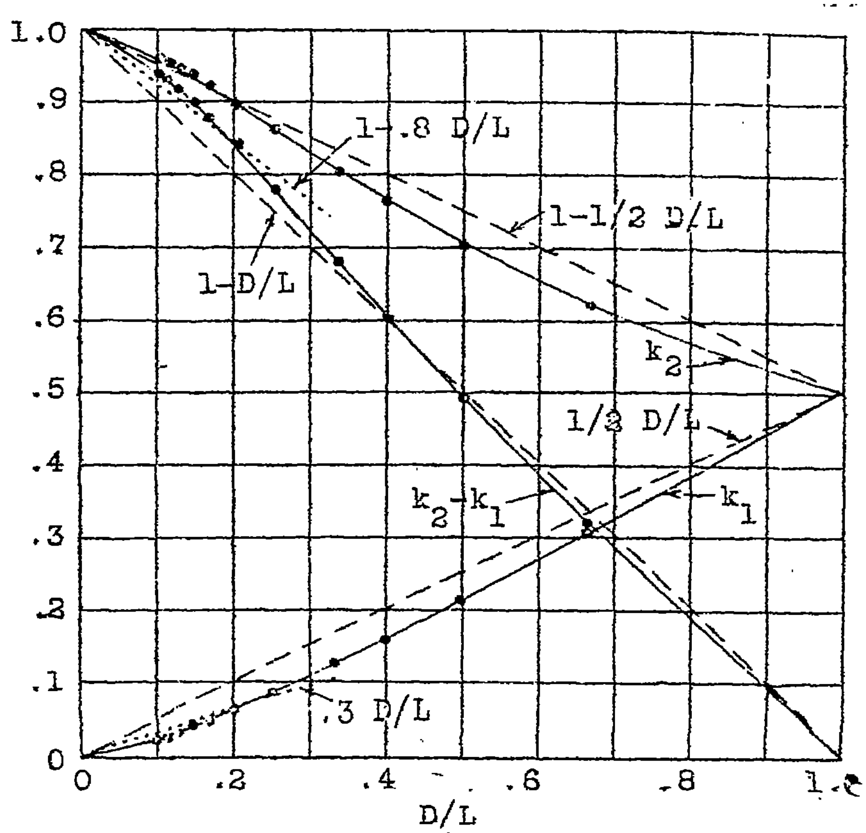


Fig.10

Figs.9 and 10

L/D	D/L	k_2	$1-\frac{1D}{2L}$	Difference	k_1	$\frac{1}{2}\frac{D}{L}$	Difference	k_2-k_1	$1-\frac{D}{L}$	Difference
∞	0.0000	1.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
9.97	0.1003	0.960	0.950	0.010	0.021	0.050	−0.029	0.939	0.900	0.039
9.02	0.1109	0.954	0.945	0.009	0.024	0.055	−0.031	0.930	0.889	0.041
8.01	0.1248	0.945	0.938	0.007	0.029	0.062	−0.033	0.916	0.875	0.041
6.97	0.1435	0.933	0.928	0.005	0.036	0.072	−0.036	0.897	0.857	0.040
6.01	0.1664	0.918	0.917	+0.001	0.045	0.083	−0.038	0.873	0.834	0.039
4.99	0.2004	0.895	0.900	−0.005	0.059	0.100	−0.041	0.836	0.800	0.036
3.99	0.2506	0.860	0.875	−0.015	0.082	0.125	−0.043	0.778	0.749	0.029
2.99	0.3344	0.803	0.833	−0.030	0.122	0.167	−0.045	0.681	0.666	0.015
2.51	0.3984	0.763	0.801	−0.038	0.156	0.199	−0.043	0.607	0.602	+0.005
2.00	0.5000	0.702	0.750	−0.048	0.209	0.250	−0.041	0.493	0.500	−0.007
1.50	0.6667	0.621	0.667	−0.046	0.305	0.333	−0.028	0.316	0.333	−0.017
1.00	1.0000	0.500	0.500	0.000	0.500	0.500	−0.000	0.000	0.000	0.000

Approximate values of Lamb's coefficients k_1, k_2 and $k_2 - k_1$ for prolate spheroid. Error less than 0.05 part of volume over whole range.



L/D	D/L	k_2	$1-\frac{1}{2}$ D/L	Difference	k_1	0.3 D/L	Difference	k_2-k_1	1-0.8 D/L	Difference
∞	0.0000	1.000	1.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
9.97	0.1003	0.960	0.950	0.010	0.021	0.030	-0.009	0.939	0.920	0.019
9.02	0.1109	0.954	0.945	0.009	0.024	0.033	-0.009	0.930	0.911	0.019
8.01	0.1248	0.945	0.938	0.007	0.029	0.037	-0.008	0.916	0.900	0.016
6.97	0.1435	0.933	0.928	0.005	0.036	0.043	-0.007	0.897	0.885	0.012
6.01	0.1664	0.918	0.917	+0.001	0.045	0.050	-0.005	0.873	0.867	+0.006
4.99	0.2004	0.895	0.900	-0.005	0.059	0.060	-0.001	0.836	0.840	-0.003
3.99	0.2506	0.860	0.875	-0.015	0.082	0.075	+0.007	0.778	0.800	-0.022
2.99	0.3344	0.803	0.833	-0.030	0.122	0.100	+0.022	0.681	0.732	-0.051

Approximate values of Lamb's coefficients for prolate spheroid, k_1, k_2 and $k_2 - k_1$
Between $\frac{L}{D} = 4$ and $\frac{L}{D} = \infty$. Maximum error is 0.02 of volume.