TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 140

GENERAL THEORY OF STRESSES IN RIGID AIRSHIP, ZR-1.

By W. Watters Pagon,
Member, Special Committee on Airship ZR-1,
National Advisory Committee for Aeronautics.

May, 1923.

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The following theory was submitted to the Special Committee on ZR-1 on August 3, 1922, in those parts dealing with Primary and Secondary Stresses, and on September 20, 1922, in that part dealing with the Effect of the Keel Girder. Since then minor changes have been made in typography and arrangement, and some comments added upon discussions by Professor Hovgaard, Member of Special Committee on ZR-1, and Mr. C. P. Burgess, Aeronautical Engineer, in his Design Memorandum No. 16, submitted to the Committee.

Primary Stresses was worked out as an individual check upon the general theory as then presented to the Special Committee; Secondary Stresses, as an attempt to evaluate this hitherto uncomputed portion of the total bending stresses; Effect of the Keel Girder, to determine quantitatively the errors in the assumption of plane bending as applied to this complicated steel-duralumin structure; and a study of the proper wire sizes to make plane bending possible, now incorporated herein under Primary Stresses, was submitted to the Committee on November 3, 1922, giving in tabular form the size required for each panel.

At several steps in the development of this theory there are introduced small errors of assumption, but when the whole is viewed in the light that the primary stresses aggregate in the worst case only 4150 lbs. per sq.in. in a total of 16,390, and the secondary stresses only 2760 lbs. per sq.in., it is clear that even the aggregate of these errors (if they should be cumulative) would be of small importance.

SUMMARY.

Primary Stresses.

$$s = \frac{y_L\,(M' - FJ'/A_d)}{I_L + nI'_d - nJ'^2/A_d}$$

$$t = n\cos^2\Phi\,\frac{s}{y_L}\left(y_m - \frac{J'}{A_d}\cos\theta\,\tan\Phi\right) + \frac{F}{A_d}\cos\theta\,\sin\Phi\,\cos\Phi$$

where s = unit stress in any longitudinal, (tension if +)
t = unit stress in any diagonal, (tension if +)
M' = bending moment at center of panel between two main transverses, (moment producing tension on top longit. and compression on keel being +)
F = shear in any panel, (+ if down on left and up on right)
y_L = distance from neutral axis to any longitudinal (+ above neut. axis)
y_m = distance from neutral axis to centre of any diagonal
θ = angle between any panel and vertical plane
Φ = angle between any shear wire and its adjacent longitudinals (+ if counterclockwise measured from longitudinal to wire)
n = ratio of moduli of elasticity of wires and longitudinals

$$= \frac{E_d}{E_L} = \frac{E\ (\text{diag.})}{E\ (\text{long.})} = \frac{E\ (\text{steel})}{E\ (\text{duralumin})}$$

a_L = sectional area of any longitudinal
a_d = sectional area of any diagonal wire
a_f = sectional area of fictitious bar $= na_d\cos^3\Phi$
L = distance between two main transverse frames
$nJ' = n\Sigma a_d y_m \cos\theta\,\sin\Phi\,\cos^2\Phi = \Sigma a_f y_m \cos\theta\,\tan\Phi = \Sigma a_f y_m h'/L$
$nA_d = n\Sigma a_d \cos^2\theta\,\sin^2\Phi\,\cos\Phi = \Sigma a_f \cos^2\theta\,\tan^2\Phi = \Sigma a_f h'^2/L^2$
h = length of any member of transverse frame, or i.e. width of any panel
h' = vertical projection of $h = h\cos\theta$, (the sign of h' being the same as that of its subtended angle, Φ)
$I_L = \Sigma\ a_L y_L^2$
$I_d' = \Sigma\ a_d y_m^2 \cos^3\Phi \qquad nI_d' = \Sigma a_f y_m^2$

For location of neutral axis, see pages 9 and 10.

SUMMARY.

Secondary Stresses.

$I_{ee'}$ = moment of inertia of longitudinal, EE'

I_{ed} = " " " " transverse member, ED

(22) $s_{ee'} = \frac{6c}{nA_dL}\left(F' - \frac{(F' + F'')}{D_e}\right)$ = secondary stress at left of joint E

(23) $s_{ee''} = \frac{6c}{nA_dL}\left(F'' - \frac{(F'' + F')}{D_e}\right)$ = secondary stress at right of joint E

(24) $P_{ee'} = \frac{6I_{ee'}}{nA_dL^2}\left(2F' - \frac{1.5(F' + F'')}{D_e}\right)$ = secondary shear at left of joint E

(25) $P_{ee''} = \frac{6I_{ee''}}{nA_dL^2}\left(2F'' - \frac{1.5(F'' + F')}{D_e}\right)$ = secondary shear at right of joint E

The stresses s, have a sign determined by the sign of the moment and by the position of the channel considered with reference to the neutral axis of the longitudinal in question. The moments are positive if counterclockwise. The shears are positive if counterclockwise, also.

$$D_e = (2 + 1.5q) + 1.5q\cos 2b.\cos 2u_e$$

where $q = I_{ed}L/I_{ee'}h$; the angle u_e is the angle between the horizontal and the radius from the center of the neutral axis to joint E; the angle b is the angle between the transverse members CE and EG and the tangent at joint E, and is a constant for joints A, C, E, G, I, K.

All other quantities used in the discussion are the same as for the primary stresses, and where primes (') are used, other than as used in the primary stress equations, they refer to the panel to the left of the joint, and where double primes (") are used they refer to the panel to the right of the joint.

General Theory of Stresses in Rigid Airship, ZR-1.

Primary Stresses.

Assumption:- All transverse frames remain plane after bending.

Signs:- + indicates tension stress and hogging moment
− indicates compression stress and sagging moment.

The following discussion will be devoted entirely to the cylindrical amidships portion, but may easily be modified for conical segments fore and aft. Consider two adjacent transverse frames and the longitudinals and shear wires between them. See Figs. 1(a) and 1(b). Let AM and A'M' be the transverses, and in order to determine the relative deflections let AM be considered as fixed. Call the bending moment at A'M', +M, and the shear just to the right of A'M', +F. Under these forces frame A'M' will rotate through an angle $\Delta L_o/y_o$ relative to AM and will deflect downward a distance Δy(a + deflection).

Let Δy_m = deflection due to moment, of frame A'M'

Δy_f = deflection due to shear, of frame A'M'

then $\Delta y = \Delta y_m + \Delta y_f$

Since radius of curvature is large compared with L,

$$\Delta y_m / L = \tfrac{1}{2} \Delta L_o/y_o = \tfrac{1}{2} g$$

See Fig. 3.

Change of length of any diagonal =

$$= \Delta L \cos\Phi + \Delta y \cos\theta \sin\Phi$$

Since frame remains plane after bending,

$$\Delta L/y = \Delta L_o/y_o = g$$

Hence change of length of diagonal =

$$= y g \cos\Phi + \tfrac{1}{2} g L \cos\theta \sin\Phi + \Delta y_f \cos\theta \sin\Phi$$

$$= g(y\cos\Phi + \tfrac{1}{2} L \cos\theta \sin\Phi) + \Delta y_f \cos\theta \sin\Phi$$

Therefore:-

Total stress in a diagonal =

(I) $$= T = \frac{E_d a_d}{L \sec\Phi}\left[g(y\cos\Phi + \tfrac{1}{2}L\cos\theta\sin\Phi) + \Delta y_f\cos\theta\sin\Phi\right]$$

Total stress in longitudinal =

(II) $$= S = E_L a_L\ \Delta L/L = E_L a_L gy/L$$

Shear in panel = F = sum of vertical comp. of diagonal stresses

(III) $$F = \Sigma T\sin\Phi\cos\theta$$

Moment = M = sum of moments of diag. stresses about neutral axis

(IV) $$M = \Sigma Sy + \Sigma T\cos\Phi\, y$$

Solving equations I and III:

$$F = \frac{E_d g}{L}\left[\Sigma a_d y\cos^2\Phi\sin\Phi\cos\theta + \tfrac{1}{2}L\,\Sigma a_d\cos^2\theta\sin^2\Phi\cos\Phi\right] + \frac{E_d}{L}\Delta y_f\Sigma a_d\cos\Phi\cos^2\theta\sin^2\Phi$$

Call $J = \Sigma a_d y\cos^2\Phi\sin\Phi\cos\theta =$

$= \Sigma a_d y\cos^3\Phi\tan\Phi\cos\theta = \Sigma a_d y\cos^3\Phi\ h'/L$

and $A_d = \Sigma a_d\cos\Phi\cos^2\theta\sin^2\Phi =$

$= \Sigma a_d\cos^3\Phi\cos^2\theta\tan^2\Phi = \Sigma a_d\cos^3\Phi\ h'^2/L^2$

Since $\tan\Phi\cos\theta = h'/L$ (see Fig. 3) It should be observed that h' has the same sign (+ or -) as $\tan\Phi$ in this expression.

then $\frac{FL}{E_d} = g(J + \tfrac{1}{2}LA_d) + \Delta y_f A_d$

(V) and $\Delta y_f = \frac{1}{A_d}\left[\frac{FL}{E_d} - g(J + \tfrac{1}{2}LA_d)\right]$

Solving equations I, II and IV:

$$M = \frac{E_L}{L}g\ \Sigma a_L y^2 + \frac{E_d}{L}g\ \Sigma a_d y^2\cos^3\Phi + \tfrac{1}{2}E_d g\ \Sigma a_d y\cos^2\Phi\cos\theta\sin\Phi + \frac{E_d}{L}\Delta y_f\ \Sigma a_d y\cos^2\Phi\cos\theta\sin\Phi$$

Call $I_L = \Sigma a_L y^2$ and $I_d = \Sigma a_d y^2\cos^3\Phi$

Substitute equation V: then equation II:

$$\text{then} \quad g = \frac{L\,[A_d M - FJ]}{A_d(F_L I_L + E_d I_d) - E_d J^2} = \frac{SL}{E_L a_L y}$$

Call $E_d = nE_L$

$$\text{then} \quad S = \frac{a_L y\ (M - FJ/A_d)}{I_L + nI_d - nJ^2/A_d}$$ and if s = unit stress in longitudinal

$$\text{(VI)} \quad \text{then} \quad s = \frac{y\ (M - FJ/A_d)}{I_L + nI_d - nJ^2/A_d}$$

From equations I and V:

$$\frac{TL}{a_d \cos\Phi} = E_d g(y\cos\Phi + \tfrac{1}{2}L\sin\Phi\cos\theta) + \frac{E_d}{A_d}\left[\frac{FL}{E_d} - g(J + \tfrac{1}{2}LA_d)\right]\cos\theta\sin\Phi$$

$$\text{or} \quad \frac{T}{a_d\cos\Phi} = \frac{(A_d M - FJ)(y\cos\Phi - J\cos\theta\sin\Phi/A_d)}{A_d I_L + nA_d I_d - nJ^2} + \frac{F}{A_d}\cos\theta\sin\Phi$$

and if t = unit stress in a diagonal wire:

$$\text{then} \quad t = n\cos^2\Phi\, y\,\frac{A_d M - FJ}{A_d I_L + nA_d I_d - nJ^2}\left(1 - \frac{J\cos\theta\sin\Phi}{A_d\, y\cos\Phi}\right) + \frac{F}{A_d}\cos\theta\sin\Phi\cos\Phi$$

$$\text{(VII)} \qquad t = n\cos^2\Phi\, s\left(1 - \frac{J\cos\theta\sin\Phi}{A_d\, y\cos\Phi}\right) + \frac{F}{A_d}\cos\theta\sin\Phi\cos\Phi =$$

$$= ns\cos^2\Phi\left(1 - \frac{Jh'}{yA_d L}\right) + \frac{Fh'\cos^2\Phi}{A_d L}$$

These equations VI and VII are the general equations for stress in any longitudinal or diagonal, regardless of what wires may be acting or not acting. In every panel there are two symmetrical wires, one with a positive angle Φ and the other with a negative angle. Each of these will appear in the summations expressed by

I_d, A_d and J. However, in A_d, $\cos\Phi$ is always positive, and $\sin^2\Phi$ must be positive also, whatever the sign of Φ. In J, $\cos^2\Phi$ is positive but $\sin\Phi$ is both positive and negative for the two wires in the panel. However, these wires have a different y and therefore do not cancel out. In an exactly symmetrical ship, with wires acting symmetrically in upper and lower portions, the corresponding upper and lower panels would have positive and negative values of y, so for this case the term J would disappear.

If it is found upon solving for these stresses t, that certain wires have a compression greater than their initial tension before loading, then these wires must be omitted from the tabulation and the computation revised. Inasmuch as the J term in the equations VI and VII has a real value in rigid ships as designed customarily, the following simplification has been made which substitutes for the J terms a new J' term, which in general is zero when all wires are acting. The method of using the formulas would be to assume that J' is zero, and then if this proves not to be the case a revision can be made to allow for the wires not acting.

Consider the expressions J and $I_d - J^2/A_d$

Let $y_m = y_L + \frac{1}{2}h'$ = height of wires at center of frame space

where y_L = height of the corresponding longitudinal to which the wire connects at the transverse in question.

Then $y_L^2 = y_m - y_m h' + \frac{1}{4}h'^2$

and $I_d = \Sigma a_d \cos^3\Phi \, y_L^2 = I'_d - LJ' + \frac{1}{4}L^2 A_d$ where I'_d and J' correspond with I_d and J but apply to the center of the frame space instead of the longitudinal.

Also $J = \Sigma a_d y_L \cos^3 h'/L = J' - \frac{1}{2}A_d L$

(VIII)
Then $I_d - J^2/A_d = I'_d - J'^2/A_d$

(IX) and $J/A_d = J'/A_d - \frac{1}{2}L$

Then let M' be the moment at center of frame space

or $M' = M + \frac{1}{2}FL$

(X) then $s = \dfrac{y_L(M' - FJ'/A_d)}{I_L + nI_{d'} - nJ'^2/A_d}$ from equations VI, VII, VIII, and IX.

(XI) and $t = n\cos^2\Phi \; \dfrac{s}{y_L}\left(y_m - \dfrac{J'h}{A_d L}\right) + \dfrac{F\,h'\cos^2\Phi}{A_d L}$

If, now, all of the wires carry compression without going out of action, then it is clear the J' for every panel is zero (provided the two wires in the panel are equal) because the respective values of $\tan\Phi$ and $\tan(-\Phi)$ cancel each other. In such case these two equations reduce to the simple form

(XII) $$s = y_L \frac{M'}{I_L + nI'_d}$$

(XIII) $$t = n\cos^2\Phi\, s\, \frac{y_m}{y_L} + \frac{Fh'\cos^2\Phi}{A_d L}$$

If some of the wires go out of action, there will still be many panels where the terms still cancel out. It is readily seen then that J' applies only to the wires not acting, and has a value equal to minus the summation of the wires not acting. It is so small usually that it may be neglected. See comments in Design Memo #16 on this subject.

Therefore, in computing stresses, J' would be assumed to be zero and the stresses worked out by equations XII and XIII. Having solved for those which would most likely go out of action J' can readily be figured and corrections easily made to all the values previously figured, and the remaining bars computed.

Equations XII and XIII prove the equations on pages 19 and 20 of Design Memorandum #7 and also prove Professor Hovgaard's theory, but they impose on the latter a condition (which was suggested by the British investigating committee), namely that the moment in the frame space must be computed at the center of its length, and that the ordinates for the fictitious bars must be measured here also.

These equations given above depend upon the assumption that the transverses ~~remain~~ remain plane after bending (i.e. that the apices of the transverse remain in a plane, which rotates however about the neutral axis). They are general in that they cover the stresses due both to bending and shear (to the first order), and the results check against the partial results obtained in the British and Italian methods and against Professor Hovgaard's theory. They also check the British theory of the proper method for using the latter.

All values of y, y_m, y_L, y_o, etc., are measured from the neutral axis of the whole frame, not that of any part of the transverses, longitudinals, or shear wires.

Let us consider a little further the expression nI'_d:

Expanding, $nI'_d = n\,\Sigma a_d y_m^2 \cos^3\Phi$. Following Professor Hovgaard's suggestion, let us call the quantity $na_d\cos^3\Phi = a_f$, where a_f is the area of a corresponding horizontal fictitious bar having a height of y_m above the neutral axis, i.e. located at mid-height

(XIV) of the panel. Then it is easily seen that $nI'_d = \Sigma a_f y_m^2$ which is simply the moment of inertia of these fictitious bars.

In a similar way it is seen that

(XV) $nJ' = \Sigma a_f h' y_m$ also $nA_d = \Sigma a_f h'^2/L^2$ and $nI_d = \Sigma a_f y^2$

The term A_d is a shearing section modulus.

Neutral Axis.

In the foregoing all dimensions y have been measured to the neutral axis, but the position of this axis has not been determined. For the longitudinals alone, it would of course be at their center of gravity; for the wires alone, at their center of gravity; but for the whole aggregation it is not yet clearly defined. Equate all horizontal components (i.e. components parallel to the fore and aft axis of the ship) of stresses in wires and longitudinals to zero. Then,

$$\Sigma S + \Sigma T\cos\Phi = 0$$

(XVI) and $$\frac{M'-FJ'/A_d}{I_L+nI'_d-nJ'^2/A_d}\left(\Sigma a_L y_L + n\,\Sigma a_d y_m \cos^3\Phi - n\frac{J'G}{A_d}\right) + \frac{FG}{A_d} = 0$$

[where $G = \Sigma a_d \cos^2\Phi \cos\theta \sin\Phi$]

When all wires are acting, and the wires of each pair are equal, then G must be zero. Therefore the last two terms are zero,

(XVII) and $$\Sigma a_L y_L + n\,\Sigma a_d y_m \cos^3\Phi = 0.$$

Of these, the first is the statical moment of the longitudinals and the second that of the fictitious bars about the neutral axis, therefore the neutral axis must lie at the center of gravity of the total assembly of bars and wires. When some of the wires are not acting then G will not be zero, but will have a term in the summation for every wire out of action. Substituting a_f and h' as above, then $nG = \Sigma a_f h'/L$ and because the h' in these panels is small the value of G will be small.

Inasmuch as equation XVI does not readily lend itself to solution in simple form, it will be assumed that G is so small that it can be neglected (this error being quite small). Then omitting from the second term those wires which do not act,

$$\Sigma a_L y_L + n\,\Sigma' a_d y_m \cos^3\Phi = 0 \qquad \text{(XVIII)}$$

where Σ' represents the partisl summation.

Now let $I_w = I_L + nI_d$ for all wires that are acting, taken about the center of gravity of longitudinals and all wires

and $I_p = I_L + nI_d$ for all wires that are acting, taken about the proper neutral axis for that condition.

Then since the moment of inertia about two parallel axes differs by an amount equal to the area multiplied by the square of the distance x, between these axes:

$$\text{(XIX)} \qquad \begin{aligned} I_w &= I_p - x^2(\Sigma a_L + n\,\Sigma' a_d \cos^3\Phi) \\ &= I_p - x^2(\Sigma a_L + \Sigma' a_f) \end{aligned}$$

Substituting in equation XVIII the values $\overset{N}{y_L} + x = y_L$ and $y_m^n + x = y_m$ there results,

$$\text{(XX)} \qquad x = \frac{\Sigma a_L y_L + \Sigma' a_f y_m}{\Sigma a_L + \Sigma' a_f}$$

Applying equation XVIII to this numerator, there remains only the statical moment of the wires which are out of action, and the term is minus. The new form of the numerator is merely this statical moment $-\Sigma'' a_f y_m$

and $$x = -\frac{\Sigma'' a_f y_m}{\Sigma a_L + \Sigma' a_f}$$

Check on the Foregoing Theory.

As a check on the theory just outlined, two methods of testing it were applied: (1) by equating the wire stresses due to vertical shear with those due to longitudinal shear; (2) by investigating the condition of equilibrium around each joint.

Longitudinal Shear.

Consider the forces acting on a single transverse frame above a horizontal section cut through any panel (See Fig. 4). The forces replacing cut bars will be those on all longitudinals on each side of the transverse, together with all wires in the panels above the one cut. Then in addition will be the four wires in the cut panel, one on each side of the frame at each side of the airship. Let all letters having subscripts be those in the left hand panel corresponding to similar unmarked letter in the right hand panel. Partial summations are designated Σ_p and include only those bars which are in the panels above the cut panel.

$$\Sigma_p s a_L + \Sigma_p t a_d \cos\Phi + 2f$$

$$-\Sigma_p s' a_L - \Sigma_p t' a_d \cos\Phi - 2f' = 0$$

Multiply and divide all terms of the summations of s and s' by y. Then since s/y_L is a constant it may be factored out, and

$$\Sigma_p s a_L - \Sigma_p s' a_L = \left(\frac{s}{y_L} - \frac{s'}{y_L}\right) \Sigma_p a_L y_L$$

From equation XI,

$$\Sigma_p t a_d \cos\Phi - \Sigma_p t' a_d \cos\Phi = \frac{s}{y_L}\left(\Sigma_p n a_d \cos^3\Phi\, y_m - \frac{J'}{A_d} \Sigma_p n a_d \cos^3\Phi \cos\theta \tan\Phi \right)$$

$$- \frac{s'}{y_L}\left(\Sigma_p n a_d \cos^3\Phi\, y_m - \frac{J'}{A_d} \Sigma_p n a_d \cos^3\Phi \cos\theta \tan\Phi \right)$$

$$+ \frac{F}{A_d} \Sigma_p \cos^3\Phi \cos\theta \tan\Phi - \frac{F'}{A_d} \Sigma_p a_d \cos^3\Phi \cos\theta \tan\Phi \,)$$

(XXI) Thus, $$2(f' - f) = \frac{s}{y_L}\left(Q - \frac{J'}{A_d} \Sigma_p a_f \cos\theta \tan\Phi\right) - \frac{s'}{y_L}\left(Q - \frac{J'}{A_d} \Sigma_p a_f \cos\theta \tan\Phi\right)$$

$$+ \frac{F}{nA_d} \Sigma_p a_f \cos\theta \tan\Phi - \frac{F'}{aA_d} \Sigma_p a_f \cos\theta \tan\Phi$$

where $Q = \Sigma_p a_L y_L + \Sigma a_f y_m$

But, from equation XI again,

$$2(f' - f) = 2(T'\cos\Phi\, a_d - T\cos\Phi\, a_d) = 2a_f \frac{s'}{y_L}\left(y_m - \frac{J'_1}{A_d}\cos\theta\tan\Phi\right)$$

$$-2a_f\frac{s}{y_L}\left(y_m - \frac{J'}{A_d}\cos\theta\tan\Phi\right) + 2\frac{F'}{nA_d}a_f\cos\theta\tan\Phi - 2\frac{F}{nA_d}a_f\cos\theta\tan\Phi$$

Subtracting the second equation from the first will give a difference, which, if the two values of $f' - f$ obtained from longitudinal shear and from vertical shear are identical, will reduce to zero.

(XXII) Thus, difference $= \frac{s}{y_L}\left(Q + 2a_f y_m - \frac{J'}{A_d}\left(\Sigma_p a_f\tan\Phi\cos\theta + 2a_f\cos\theta\tan\Phi\right)\right)$

$$-\frac{s'}{y_L}\left(Q + 2a_f^1 y_m - \frac{J'_1}{A_d}\left(\Sigma_p a_f\tan\Phi\cos\theta + 2a_f^1\cos\theta\tan\Phi\right)\right)$$

$$+\frac{F}{nA_d}\left(\Sigma_p a_f\cos\theta\tan\Phi + 2a_f\cos\theta\tan\Phi\right)$$

$$-\frac{F'}{nA_d}\left(\Sigma_p a_f\cos\theta\tan\Phi + 2a_f'\cos\theta\tan\Phi\right)$$

If wires of each pair in a panel are equal, then $\Sigma a_f\cos\theta\tan\Phi$ is zero because Φ is both + and - ; but this is true only if all wires are acting. In such case J' and J'_1 are both zero. Therefore, applying equation XI,

$$\text{Difference} = \frac{M' - M'_1}{I}\left(\Sigma_p a_L y_L + \Sigma_p a_f y_m + 2a_f y_m\right)$$

$$-\frac{F + F'}{nA_d}(2a_f\cos\theta\tan\Phi') \quad \text{since } \tan\Phi' \text{ is } + ,\ \tan\Phi \text{ is } -$$

where $I = I_L + nI_d$. Now let the parenthesis of the moment term be Q' where $Q' = Q + 2a_f y_m$. Then since $M' = M'_1 + \frac{1}{2}L(F' + F)$

(XXIII) $\text{Difference} = (F' + F)\left(LQ'/2I - \frac{2a_f\cos\theta\tan\Phi'}{nA_d}\right)$

Therefore the parenthesis must be zero, and this term has in it only terms relating to the structure itself , without regard to the bending moments or shears.

(XXIV) Thus, $\frac{LQ'}{2I} - \frac{2a_f\cos\theta\tan\Phi}{nA_d} = 0$

This formula applies to primary and secondary wires, and if we let a_f^p be the area of a fictitious bar corresponding to the primary wires, and a_f^s be the area of a fictitious bar corresponding to the secondary wires; and if we note that a horizontal section one panel length L, long will cut two primary and four secondary wires on each side of the ship; then the formula becomes

$$\text{(XXV)} \qquad \frac{LQ'}{2I} - \frac{2a_f^p \cos\theta \tan \Phi_p}{nA_d} - \frac{4a_f^s \cos\theta \tan \Phi_s}{nA_d} = 0$$

Noting that Φ_p and Φ_s have different slopes,

then, $\cos\theta \tan\Phi_p = h'/L$ and $\cos\theta \tan\Phi_s = 2h'/L$

(XXVI) and, $a_f^p + 4a_f^s = \dfrac{nA_d L^2}{4I} \dfrac{Q'}{h'}$

From this equation XXVI it is possible to compute the sizes of fictitious bars, that is, the sizes of wires both primary and secondary, that must be used in the ship to cause it to deform in accordance with the plane bending theory. The equation contains the terms A_d and I which depend upon the wire sizes, the first entirely and the latter to a slight extent, therefore it is necessary to have some assumed values of wire sizes in order to apply the equation. The relative sizes of wires, one to another, are dependent only upon the statical moment Q' of the cross sectional area above the panel containing the wires under consideration, and the height h', of this panel. The following study shows that such a revision of the sizes relative to one another will not change A_d.

Let A_d be the shearing modulus for the assumed wire sizes, and A_d^c the same modulus computed from the revised sizes.

Then $A_d^c n = \Sigma a_f^p h'^2/L^2 + \Sigma a_f^s (2h')^2/L^2 = \frac{1}{L^2} \Sigma (a_f^p + 4a_f^s) h'^2$

Substituting equation XXVI above in this equation

then, $A_d^c = \dfrac{1}{nL^2} \Sigma \dfrac{na_d L^2}{4I} \cdot \dfrac{Q'}{h'} \cdot h'^2 = A_d \dfrac{\Sigma Q'h'}{4I}$

(Note that in deriving equation XXIII, the minus signs were eliminated for h' thus, the terms of this summation do not cancel.)

Since $Q' = \Sigma_p ay$ for all members above any plane of shear

then, $\Sigma Q'h' = a_1 y_1 (h_1' + h_2' + h_3' + \text{etc.}) + ay_2 (h_2' + h_3' + \text{etc.}) + \text{etc.}$

$= a_1 y_1^2 + a_2 y_2^2 + \text{etc.} = \Sigma ay^2 = 4I$ because $Q'h'$ appears for each main wire and there are four wires per panel (two on each side). Therefore, $A_d^c = A_d$; which proves the statement at the beginning of this paragraph, that revision of the wire sizes by the formula given will not change A_d and the other computations which depend upon it.

For the panel MN of ZR-1 as designed, since there are four pairs of wires in place of the single pair in other panels,

the formula becomes $\frac{LQ'}{2I} - \frac{8a_f \cos\theta \tan\Phi_{mn}}{nA_d} = 0$

But, $\cos\theta \tan\Phi_{mn} = 4h'/L$ therefore, $16a_f^{mn} = \frac{na_dL^2}{4I} \cdot \frac{Q'}{h'}$.

The attached Table I shows a computation of the theoretical wire sizes for ZR-1. In making this computation the average value of wire size in each panel (for example, the average of wires in AB and BC, and the average of EF and FG, etc.) has been obtained. Due to the fact that intermediate longitudinal and fictitious bars occur at center of panel, the main primary shear wires would change size at the intermediate longitudinal, although in the actual ship the size remains constant. By taking the Q' term to include one-half only of the ay term for the intermediate longitudinal and fictitious bars in the panel which is being considered, this average value for a_f^p is obtained. It will be noted in the last column of the table that the results are checked up to determine their correctness, and an error of 5 per cent appears, which is due probably to the fact that $\Sigma Q'$ is not quite zero, or in other words, that the neutral axis as computed and used in the design of the ship is not exactly at the center of gravity of the cross section.

There has been much discussion of the so-called "ideal ship," which would consist merely of a symmetrical, polygonal cross section, with uniform sizes of wires and longitudinals in all panels. The polygonal cross-section would of course be regular, so that a circumscribing circle could be drawn through its apices. Let us assume that the shape is symmetrical about a vertical axis, which involves an error of only one or two per cent from the 25-sided figure customarily used for rigid ships. Then from equation XXVI,

$$a_f^p + 4a_f^s = \frac{nA_dL^2Q'}{4Ih'} = \frac{Q'}{4Ih'}\Sigma(a_f^p + 4a_f^s)h'^2 = \frac{Q'}{4Ih'}(a_f^p+4a_f^s)2 \quad \Sigma h'^2$$

since the wire sizes are constant in all panels. Therefore,

XXVII) $2Ih' = Q' \Sigma h'^2$ where the summation is for both sides of ship

ut, $h' = h\cos\theta$, $y_m = R\sin\theta$, $y_L = R\sin(\theta - a)$ (See Fig. 5).

Hence, $I = a_L R^2 \Sigma \sin^2(\theta - a) + 2a_f R^2 \Sigma \sin^2 \theta$

Let the number of panels in a quadrant be

$N = \frac{1}{4}N'$ for the whole ship,

then $\Sigma \sin^2\theta = \Sigma \cos^2\theta$, by symmetry

$\Sigma \sin^2\theta = N - \Sigma\cos^2\theta$, for one quadrant

hence, $\Sigma \sin^2\theta = \Sigma\cos^2\theta = \frac{1}{2}N$, for one quadrant $= 2N$ for the whole ship.

Thus, $I = 2NR^2(a_L + 2a_f)$. By similar proof,

$$Q' = Ra_L \Sigma_p \sin(\theta - a) + 2Ra_f \Sigma_p \sin\theta + 2a_f R\sin\theta$$

Referring to Fig. 5, $\Sigma_p h\sin\theta = c'$ and $\Sigma_p h\sin(\theta - a) = c$

Thus, $Q' = (a_L c + 2a_f c' + 2a_f h\sin\theta_e)R/h = (a_L + 2a_f)Rc/h$

And substituting these values in equation XXVII,

$$4NR^2(a_L + 2a_f)h\cos\theta_e = \frac{Rc}{h}(a_L + 2a_f)h^2 2N = 4R^2 N(a_L + 2a_f)h\cos\theta_e$$

since $c = 2R\cos\theta_e$. This being an identity, it is clear that a uniform size of wires, both primary and secondary, in all panels is the one requisite for plane bending in the ideal ship. Such a ship would of course have all longitudinals of equal size, but only slight error will be introduced if the main and secondary longitudinals are made of different section, provided each set are of uniform size. A further discussion is given under the caption "Effect of the Keel."

Table I.

Theoretical Sizes of Shear Wires from Formula $a_f^p = \frac{nA_dL^2}{4I} \times \frac{Q'}{h'} - 4a_f^s$

Longs.	Main Wires	Sec. Wires	Area of one Mem.	a_f for 4 wires	y	y_m	h'	$a_f^p h'^2$ for 4 wires	$4 \times a_f^s h'^2$ for 4 wires	$a_L y^2$ or $a_f y_m^2$	$a_L y$ or $a_f y_m$
			mm^2	mm^2	m	m	m	$mm^2 \times m^2$	$mm^2 \times m^2$	$mm^2 \times m^2$	$mm^2 \times m$
P			221		-8.25					30100	-3640
N			210		-7.35					11350	-1545
	MN		6.59	11.2		-8.80	2.91	1520		870	- 97
O			144		-10.28					15100	-1475
M			197.5		-10.26					41400	-4050
L			167		- 9.42					29600	-3150
	KM		5.27	38.6		-9.42	2.29	202		3430	- 363
K			186		- 7.97					23600	-2965
J			136		- 5.93					9580	-1610
	IK		6.59	48.2		-5.93	4.51	1010		1670	- 286
I			186		- 3.40					4300	-1265
H			136		- 0.61					100	- 161
	GI		6.59	48.2		-0.61	5.74	1590		18	- 30
		GI	3.32	10.2		-0.61	5.74		1344	2	- 59
G			186		+ 2.34					2040	+ 870
F			136		+ 5.24					7480	+1420
	GE		6.59	48.2		+5.24	5.51	1330		1320	+ 252
		GE	332	10.2		+5.24	5.51		1240	280	56
E			186		+ 7.85					22900	2940
D			136		+10.15					28000	2760
	EC		5.27	38.6		+10.15	3.95	602		4000	392
		EC	3.32	10.2		+10.15	3.95		640	1050	40
C			186		+11.80					51900	4390
B			136		+12.94					45600	3520
	CA		5.27	38.6		+12.94	1.43	79		6420	500
		CA	3.32	10.2		+12.94	1.43		84	1700	15
A			216		+13.23					38000	2860
								6333 3308	3308		
						nA_dL^2		= 9641		381890 = I	

Table 1 - Theoretical Sizes of Shear Wires from Formula $a_f^p = \frac{nA_dL^2}{4I} \times \frac{Q'}{h'} - 4a_f^s$ (Contd.)

Main Wires	Q'	$\frac{nA_dL^2}{4I} \times \frac{Q'}{h'}$	$4a_f^s$	Comp. a_f^p	Theoretical a_d^p	Actual a_d^p	Revised $a_f^p\, h'^2$ for 4 wires
	$mm^2 \times m$	mm^2	mm^2	mm^2	mm^2	mm^2	$mm^2 \times m^2$
MN	5233	11.3 ÷ 16		= 0.71	1.66	6.59	38.4
KM	12563	34.5		34.5	18.8	5.27	724
IK	18233	25.1		25.1	13.7	6.59	2100
GI	20571	22.2	10.2	12.0	6.55	6.59	1580
GE	18281	20.9	10.2	10.7	5.85	6.59	1300
EC	12881	20.5	10.2	10.3	5.63	5.27	640
CA	4878	21.5	10.2	11.3	6.18	5.27	92

6820
3308

Revised nA_dL^2 = 10128

error $\frac{10128-9641}{9641}$ = 5%

Equilibrium of a Joint.
(See Fig. 6)

Consider the forces acting on any main joint of a frame. Let subscript letter "e" denote quantities in panel below joint and "d" above joint.

For equilibrium

$$T'_e \cos\Phi_e + T'_d \cos\Phi_d + S' = T_e \cos\Phi_e + T_d \cos\Phi_d + S$$

From equation XV,

$$T'_e \cos\Phi_e = na_d \cos^3\Phi_e \frac{s'}{y_L}\left(y_{me} - \frac{J'}{A'_d}\cos\theta_e \tan\Phi_e\right) + \frac{F'}{A'_d} a_d \cos^3\Phi_e \cos\theta_e \tan\Phi_e$$

$$\tan\Phi'_e = -\tan\Phi_e \quad \text{and} \quad -\tan\Phi'_d = +\tan\Phi_d$$

Therefore,

$$(T'_e - T_e)\cos\Phi_e = a_{fe} y_{me}\left(\frac{s'-s}{y_L}\right) - a_{fe}\cos\theta_e \tan\Phi_e\left(\frac{s'J'}{y_L A'_d} + \frac{sJ'}{y_L A_d}\right)$$

$$+ \left(\frac{F'}{nA'_d} + \frac{F}{nA_d}\right) a_{fe}\cos\theta_e \tan\Phi_e$$

$$(T'_d - T_d)\cos\Phi_d = a_{fd} y_{md}\left(\frac{s'-s}{y_L}\right) - a_{fd}\cos\theta_d \tan\Phi_d\left(\frac{s'J'}{y_L A'_d} + \frac{sJ'}{y_L A_d}\right)$$

$$-\left(\frac{F'}{nA'_d} + \frac{F}{nA_d}\right) a_{fd}\cos\theta_d \tan\Phi_d$$

$$S' - S = a_L y_L \left(\frac{s'-s}{y_L}\right)$$

Adding these three equations will give an expression which must reduce to zero if the joint is to be in equilibrium.

$$\text{XXVI)} \quad \text{Sum} = \left(\frac{s'-s}{y_L}\right)(a_{fe} y_{me} + a_{fd} y_{md} + a_L y_L)$$

$$+ a_{fe}\cos\theta_e \tan\Phi_e \left(\frac{F'}{nA'_d} + \frac{F}{nA_d} - \frac{s'J'}{y_L A'_d} - \frac{sJ'}{y_L A_d}\right)$$

$$- a_{fd}\cos\theta_d \tan\Phi_d \left(\frac{F'}{nA'_d} + \frac{F}{nA_d} + \frac{s'J'}{y_L A'_d} + \frac{sJ'}{y_L A_d}\right)$$

When all wires are acting, $J' = 0$ and summations for both frame spaces are equal (i.e. $A'_d = A_d$ etc.)

Substituting these values and value of s' and s from equation X,

$$\text{XXVII)} \quad \text{Sum} = \left(\frac{M' - M}{I}\right)(a_{fe}y_{me} + a_{fd}y_{md} + a_L y_L)$$

$$+ \frac{F'+F}{nA_d}(a_{fe}\cos\theta_e\tan\Phi_e - a_{fd}\cos\theta_d\tan\Phi_d)$$

But, $M' - M = -\frac{L}{2}(F' + F)$

When a_f, a_L and Φ are constant,

$$\text{Sum} = -(F'+F)\left[\frac{L}{2I}(a_f y_{me} + a_f y_{md} + a_L y_L)\right.$$

$$\left. - \frac{a_f h}{nA_d L}(\cos\theta_e - \cos\theta_d)\right]$$

For the "ideal ship"

$$I = 2NR^2(a_L + 2a_f)$$

$$\text{XXVIII)} \quad nA_d = \Sigma a_f \cos^2\theta \frac{h^2}{L^2} = 4Na_f \frac{h^2}{L^2}$$

Then, $\text{Sum} = -L(F' + F)\left(\frac{a_f y_{me} + a_f y_{md} + a_L y_L}{4NR^2(a_L + 2a_f)} - \frac{(\cos\theta_e - \cos\theta_d)}{4Nh}\right)$

$$= -\frac{L(F' + F)}{4NR^2(a_L + 2a_f)}\left[a_f(y_{me} + y_{md} - \frac{2R^2}{h}(\cos\theta_e - \cos\theta_d)\right)$$

$$\left. + a_L(y_L - \frac{R^2}{h}(\cos\theta_e - \cos\theta_d)\right]$$

$R(\cos\theta_e - \cos\theta_d) = \frac{h}{2}(\sin\theta_e + \sin\theta_d)$ See Fig. 5.

$R\sin\theta_e = y_{me}$ $\qquad$ $R\sin\theta_d = y_{md}$

Therefore

$$\text{Sum} = -\frac{L(F' + F)}{4NR^2(a_L + 2a_f)}\left[a_f(y_{me} + y_{md} - y_{me} - y_{md}) + a_L\left(y_L - \frac{y_{me} + y_{md}}{2}\right)\right]$$

$$= \text{zero.}$$

This further check on the theory shows that when the longitudinals and the shear wires are of constant section from panel to panel, i.e. in the ideal ship, the assumption of planar bending is completely justified. Conversely, it is clear that even though the longitudinals be averaged and used thus (which probably introduces only a small error for all longitudinals above the ones at L and an appreciable error for the keel girders), still unless a_f for the two adjacent panels is constant the quantity derived above does not reduce to zero. Therefore, the planar theory cannot be true for a joint where the wire size changes, though it still holds for all panels below such joint and above as well.

General Theory of Stresses in Rigid Airship, ZR-1.

Effect of Keel Girder.

There has been much discussion of the effect of heavy keel longitudinals M and M' and of the apex girder N, of the keel corridor. These three longitudinals are connected by diagonal shear wiring to form a triangular girder in themselves, and of course the shear wires transmit longitudinal shear as well, so that the N girder also contributes to the general strength of the ship.

In order to analyze the stresses of the combination of these with the so-called "ideal ship" consisting of a group of girders at the apices of a regular polygon of 24 sides (which is substantially the same as one of 25 sides, if the two M girders are looked upon as one and the same, but split apart and connected together by transverse ties - since by symmetry there can be no shear between them), let us look at the history of the type. Starting with merely an elongated gas bag with a single car suspended by a plurality of inclined suspenders, the first logical step was the introduction of a keel, or distributing girder, whose function, like that of the stiffering truss of a cable suspension bridge, is to transform certain loads concentrated at a few points into a uniform set of loads on the suspenders. In this case all shears and bending moments produced in bringing these two sets of forces into equilibrium were carried by the keel girder. Next the suspenders can be looked upon as a set of rings encircling the gas-filled envelope and carrying loads to it in much the same way as the rings of a telephone cable encircle and transmit loads to the "messenger" cable. The next logical step is to provide shear wiring between these rings, or transverses, so that they form in themselves a rigid structure, connected to the rigid keel by a set of rigid vertical suspenders, so that both structures deflect identically at the transverse frames. This type of ship was the earliest form of rigid. The last step was to place the keel within the circumscribing circle of the "ideal ship," but structurally this has no significance other than in the details of the computations and in the diminished effect of these keel members due to their lessened distance from the neutral axis.

For simplicity of conception let us adhere to the earlier type with the suspended keel, but at the same time we will hold to the true dimensions of the later type of "internal" keel. For the present, also, let us assume that at all transverses the entire structure is so held together by the transverses (assumed to be incapable of distortion of shape in their own planes) that every part of the compound ship deflects alike at any one transverse; holding for later discussion Mr. Burgess' proof that the intermediate transverses are incapable of exerting sufficient force on the keel to do this effectively.

We can then, in the light of the previous discussion, think of the compound ship as being composed of three structures, all rigidly connected at regular intervals so as to deflect identically at those points; the three structures being,

(a) the "ideal ship," composed of 24 sides and 24 longitudinals A to M, the two M girders being joined into one girder of cross section identical with that of the others, (A to L);
(b) the keel, actually within the same but imagined as though suspended below it, and composed of the N girder and of two partial M girders having each a cross section equal to the actual section minus that portion which is considered to be contained within the M girder of the ideal ship;
(c) a girder, somewhat of the order of a reinforced concrete girder, consisting of an upper portion consisting of the girders of the ideal ship (simulating the concrete) and a lower portion consisting of the keel (simulating the steel reinforcement), both of which parts are connected by shear wires to transmit the longitudinal shear.

Let the subscript "I" denote members and quantities that deal with the ideal ship; "k", those that deal with the keel; "g" those that deal with the girder of case (c) above. Where no subscript is used it is to be understood that the quantity refers to the whole aggregation or to the summation of the others.

Thus, $I = I_i + I_k + I_g$ referring to the moments of inertia

$w = w_i + w_k + w_g$ referring to the proportional loadings.

If any joint of the compound ship deflect a distance Δ

then $\Delta = \dfrac{kL^n w_i}{E_i I_i} = \dfrac{kL^n w_k}{E_k I_k} = \dfrac{kL^n w_g}{E_g I_g}$ where $k = 1/48$ or $5/384$ etc.
$n = 3$ or any other number.

Because all three structures are similarly constructed of wires and duralumin longitudinals, $E_i = E_k = E_g$

Thus, $\dfrac{w_i}{I_i} = \dfrac{w_k}{I_k} = \dfrac{w_g}{I_g} = \dfrac{w}{I}$

and $w_i = \dfrac{w}{I} I_i \quad w_k = \dfrac{w}{I} I_k \quad w_g = \dfrac{w}{I} I_g$

Also $M = cL^m w \quad M_i = cL^m w_i \quad M_k = cL^m w_k \quad M_g = cL^m w_g$

where "c" is a constant, such as 1/8 or 1/4

m is 2 or 1 or other number.

Thus, $M_i = \dfrac{M}{w} w_i = \dfrac{M}{I} I_i \quad M_k = \dfrac{M}{w} w_k = \dfrac{M}{I} I_k \quad M_g = \dfrac{M}{w} w_g = \dfrac{M}{I} I_g$

Let, in general, f be the unit stress at distance y from neutral axis

Then, $f_i = \dfrac{M_i}{I_i} y_i = \dfrac{M}{I} y_i \quad f_k = \dfrac{M_k}{I_k} y_k = \dfrac{M}{I} y_k \quad f_g = \dfrac{M_g}{I_g} y_g = \dfrac{M}{I} y_g$

y_i being measured from center of gravity of wires and longitudinals of the ideal ship; y_k from center of gravity of keel; y_g from center of gravity of wires, longitudinals of ideal ship, and also the girder N and the "girder-portion" of the two M girders.

Let A_m^i = area of two girders M included in ideal ship

A_m^k = area of two girders M included in keel alone

Then $A_m^i + A_m^k = A_m$ = area of two girders M, total. Of course the area of the two M's which enter into the two flanges of the girder of case (c) are the same areas; namely, A_m^i being that part of the area which enters into the total area of the top flange, and A_m^k being that part which enters into the total area of the bottom flange. That part of the area which is in the top flange would receive tensile stress for hogging moment, whereas when it is considered part of the ideal ship it will receive compression; that part of the area which is in the bottom flange would receive compression for hogging moment, and when it is considered part of the keel it will likewise receive compression.

Therefore, to find the total stress in the members M we must find the total stress on each imaginary part by multiplying its area by its respective stress and then adding the two results. Thus,

$$-F = f_k A_m^k + f_g A_m^k + f_i A_m^i - f_g' A_m^i = \frac{M}{I}(A_m^k(y_k+y_g) + A_m^i(y_i - y_g'))$$

In this equation y_k is the distance from c. of g. of keel to M; y_g is the distance from c. of g. of girder (which is the same as the c. of g. of the compound ship) to c. of g. of the keel; y_i is the distance from c. of g. of ideal ship to M; and finally y_g' is the distance from c. of g. of girder (or compound ship) to c. of g. of ideal ship, or in other words it is the amount which the center of gravity is displaced in the ideal ship if the keel girder areas are taken into the calculation. Therefore,

$y_g + y_k$ = distance from M to c. of g. of entire compound ship
$y_i - y_g'$ = distance from M to c. of g. of entire compound ship.

Therefore the equation above can be written

$$-F = \frac{M}{I}y(A_m^k + A_m^i) = \frac{M}{I}yA_m$$

and dividing through by A_m, then $f_c = \frac{M}{I}y$ where f_c is the compression per sq. in. over the whole section of the M girders and y is the distance from M to the c. of g. of the compound ship.

In other words, the actual stress developed in M is identically the same as though the compound ship were figured, using total moment of inertia, total moment, and distances y measured from neutral axis through the c. of g. of the whole ship. From which it follows that addition of a keel to the ideal ship does not invalidate the theory of plane bending, provided the shear wires are adequate.

Considering now the N girder, at the top of the keel, it is clear that none of its cross section enters into the ideal ship, but the whole section enters into both keel and girder. Further, it is clear that for a hogging moment, its stress when acting as top member of the keel is tensile, whereas its stress as part of the bottom flange of the compound girder is compression. Therefore,

$$-F = f_g A_n - f_k A_n = A_n M(y_g - y_k)/I = A_n My/I$$

where y is the distance from N to the c. of g. of the "girder" or of the compound ship. Also, as before, $f_c = My/I$, where f_c is the compression over the whole section of the N girder. Which again is in accordance with the plane bending theory.

Mr. Burgess has shown that the intermediate transverses are not of sufficient rigidity to cause the keel to move upward equally with the other longitudinals, although undoubtedly the tension of the gas cell wires and of the envelope both tend to cause uplift. However, a correction can quite readily be made for this divergence from the assumption of identical deflections, by applying the theorem of three moments; and moreover, the stresses are secondary ones and not of great magnitude.

Let us now consider shearing stresses and deformations. There will be no longitudinal shear in the panels of the ship due to the bending of the keel within itself, but the wires connecting N with M and M' will transmit shear so as to permit N to act with the ship in bending more or less completely, as just outlined. In the ideal ship the longitudinal shear in a panel length on the wires just above M and M' will equal

$$L\frac{d}{dL}f_i A_m^i = L\frac{d}{dL}M_i y_i A_m^i/I_i = L\frac{dM}{dL}y_i A_m^i/I = LSQ_k^i/I$$

where S = shear at the frame space in question, and Q_k^i = statical moment of A_m^i about c. of g. of compound ship.

In the girder of case (c) the longitudinal shear in a panel length on the wires just above M and M' will equal

$$L\frac{d(\text{flge. str.})}{dL} = L\frac{df_g}{dL}(A_m^k + A_n) = L\frac{dM_g y_g}{dLI_g}(A_m^k + A_n) = LSQ_k^g/I$$

where S =shear, as before, and Q_k^g = moment of keel, i.e. of $A_m^k + A_n$, about c. of g. of compound ship.

Therefore, the total longitudinal shear in a panel length on the wires just above M and M' will equal the sum of these

$= LSQ_k^i/I + LSQ_k^g/I = LSQ_k/I$ where Q_k = statical moment of M, M' and N about the c. of g. of compound ship. This again is the same result as would be obtained by computing the longitudinal shear in the compound ship in the usual way by the plane bending theory.

The discussion, however, thus far has pre-supposed that for the girder of case (c) the web connected the c. of g. of the bottom flange to that of the top flange, directly; and from this c. of g. the stress would radiate to each member of that flange. In the actual compound ship, however, it is otherwise. Shear from the keel accumulates at M and M' and then travels circumferentially to L and L', then to K and K', etc., up to A. Let us therefore discuss the modifications involved by this path, but in so doing let us only discuss "girder" stresses, as superposed on the already existing stresses of the ideal ship which have been previously discussed in the first part of this memorandum.

If the upper flange, i.e. the ideal ship, has all longitudinals equal, of number n, then each longitudinal will receive one nth part of the shear LSQ_k^g/I, and the longitudinal shear will diminish upward equally from panel to panel by two nths. If, as in the actual case, each main longitudinal have one mth part and each intermediate one pth part of the flange area, then the longitudinal shear will diminish upward two mths and two pths alternately. In the first case the reduction of shear is linear <u>circumferentially</u>, since all panels have an equal height h, and in the latter, while the reduction is zig-zag, it will be linear every two panels, and can therefore be considered circumferentially linear, very approximately. If the wires whether main or counter, in all panels are equal in cross section, then their elongation and the joint deflections due to longitudinal shear will be linear <u>circumferentially</u>, provided that the counter-shear wires act either in <u>all</u> panels or in <u>none</u>. But a circumferentially linear deflection, when projected on the diameter of the ship, is no longer linear, therefore the shear deflection due to that part of the shear wire stresses which is caused by shear from the "girder" action, is not planar. It is, in fact, the curve obtained by projecting a helix of constant pitch on a plane through its axis. However, superposition of non-planar shear deflections upon planar bending ones does not in itself invalidate the bending theory.

Let us now consider the effect of this method of transmitting the "girder" shear around the perimeter upon the longitudinals themselves. If all counter-shear wires are acting, then each tension wire will transmit its stress to the adjacent compression wire, less the nth part that passes into the longitudinal at the joint; and vice versa. Therefore, all longitudinals above M M' will get an equal tension (for hogging moment) and a plane section will deflect as a plane moved parallel to itself; but N, being compression, will not deflect in the same plane, and M and M' will move an intermediate distance, having tension on that part of their area which is in the top flange and compression on the remainder. If, on the other hand, the counter wires do not act, then the tension of each shear wire must be carried in compression back through the longitudinal just above it, in order to transmit shear to the next shear wire. This will cause stresses in the longitudinals varying linearly around the circumference, which cannot therefore be planar.

Hence, while if girder stresses could be ideally distributed to the c. of g. of each flange, the bending and shear deflections would be planar, for constant wire size, still this cannot be done, and therefore the actual distribution of these stresses will be somewhat out of a plane. The resultant error, however, is small and the plane bending theory is not many per cent in error as regards the stresses in longitudinal girders.

Inasmuch as the deviations from plane bending are due only to shear deflections, the resulting errors will be only at frame spaces having high shear, therefore it is to be expected that at the concentrations due to the suspended cars, or at a space where there is reversal of shear, would be the only regions of appreciable error, which is true of any type of beam, in general, carrying concentrated loads. At such points the effect of excessive shear deflections will be to diminish the rate of change of stress in N and to a less extent M and M'.

In all the above it has been assumed that the wire sizes are constant in all panels. If this is not true (and it is not in rigid airships as actually built), there will be introduced an error of the same order as that in the ideal ship.

In the light of this discussion it would seem to be advisable to increase the wire sizes. There are objections to this, however, due to the condition of the hull when the gas bags are partially deflated.

General Theory of Stresses in Rigid Airship ZR-1.

Secondary Stresses.

Consider a stiff member acted on by forces and moments as shown by Fig. 8. Let moments and rotations in counterclockwise direction be positive, also deflections.

Change of slope $= \alpha_b - \alpha_a = \int_b^a M\frac{dx}{EI}$ (Origin at B)

Total deflection $= \Delta_{ba} = \alpha_a L + \int_b^a Mx\frac{dx}{EI}$

$$M = M_{ba} + P_b x$$

$$\therefore \alpha_b - \alpha_a = \int_o^L M_{ba}\frac{dx}{EI} + \int_o^L P_b x\frac{dx}{EI} = \frac{1}{EI}\left(M_{ba}L + \frac{P_b L^2}{2}\right)$$

$$\Delta_{ba} = \alpha_a L + \int_o^L M_{ba}x\frac{dx}{EI} + \int_o^L P_b x^2\frac{dx}{EI} = \alpha_a L + \frac{1}{EI}\left(\frac{M_{ba}L^2}{2} + \frac{P_b L^3}{3}\right)$$

$$M_{ab} = M_{ba} + P_b L$$

$$\therefore \Delta_{ba} = \alpha_a L + \frac{L^2}{6EI}\left[3M_{ab} - P_b L\right]$$

(1) $$M_{ab} = \frac{2EI}{L}\left(\frac{\Delta_{ba}}{L} - \alpha_a\right) + \frac{P_b L}{3}$$

Substituting $M_{ab} - P_b L$ for M_{ba} and value of $P_b L$ from eq. 1 in equation for $\alpha_b - \alpha_a$ we have

$$\alpha_b - \alpha_a = \frac{1}{EI}\left(M_{ab}L - \frac{3}{2}M_{ab}L\right) + 3\left(\frac{\Delta_{ba}}{L} - \alpha_a\right)$$

(2) $$\therefore M_{ab} = \frac{2EI}{L}\left(\frac{3\Delta_{ba}}{L} - 2\alpha_a - \alpha_b\right)$$

See Fig. 9.

Consider a transverse member ED of length h making an angle θ_{ed} with vertical. Assume joint E to rotate in vertical plane ZEX through an angle α_e, and joint D to deflect longitudinally a distance Δ_{de} and rotate in vertical plane parallel to ZEX through angle α_d. Since deflection of one frame relative to another, which produces secondary stresses and rotation of joints, is vertical, rotations of joints laterally are of second order and hence are neglected. See Appendix I.

In the plane DEE' the rotation of D is $\alpha_d \cos\theta_{ed}$ and of E is $\alpha_e \cos\theta_{ed}$.

The bending moment at E of ED in plane DEE' is therefore given by equation (2) as follows:

(3) $$M^b_{ed} = \frac{2EI_{de}}{h}\left(\frac{3\Delta_{de}}{h} - 2\alpha_e \cos\theta_{ed} - \alpha_d \cos\theta_{ed}\right)$$

In plane perpendicular to DEE' joint E rotates through angle $\alpha_e \sin\theta_{ed}$ and D rotates $\alpha_d \sin\theta_{ed}$ producing in ED a counterclockwise torsional moment at

(3a) $$E = M^t_{ed} = \frac{(\alpha_d - \alpha_e)\sin\theta_{ed} E_T I_T}{h}$$

The components of these moments about axis EY are

$M^b_{ed}\cos\theta_{ed}$ and $M^t_{ed}\sin\theta_{ed}$.

See Fig. 10.

Equations for member EF are similar.

(4) $$M_{ee'} = \frac{2EI_{ee'}}{L}\left(\frac{\Delta_{ee'}}{L} - \alpha_e\right) + \frac{P_{ee'}L}{3} \quad \text{from eq. 1.}$$

(5) $$M_{ee'} = \frac{2EI_{ee'}}{L}\left(\frac{3\Delta_{ee'}}{L} - 2\alpha_e - \alpha_{e'}\right) \quad \text{from eq. 2.}$$

(6) $$M_{ee''} = \frac{2EI_{ee''}}{L}\left(\frac{\Delta_{ee''}}{L} - \alpha_e\right) + \frac{P_{ee''}L}{3} \quad \text{from eq. 1}$$

(7) $$M_{ee''} \quad \frac{2EI_{ee''}}{L}\left(\frac{3\Delta_{ee''}}{L} - 2\alpha_e - \alpha_{e''}\right) \quad \text{from eq. 2.}$$

For equilibrium, Σ moments about axis EY = 0

Therefore,

(8) $$M_{ee'}+M_{ee''}+M^{b}_{ed}\cos\theta_{ed}+M^{t}_{ed}\sin\theta_{ed}+M^{b}_{ef}\cos\theta_{ef}+M^{t}_{ef}\sin\theta_{ef}=0$$

Assume that points D and F do not deflect out of plane of transverse frame (see p. 1),

then Δ_{de} and $\Delta_{fe} = 0$

Substituting in (8) values of moments given by (3), (3a), (5) and (7), and for convenience let

$$\frac{\Delta_{ee'}}{L} = R_{e'} \quad \text{and} \quad \frac{\Delta_{ee''}}{L} = R_{e''}, \quad \text{then}$$

$$\frac{2EI}{L}(3R_{e'} + 3R_{e''} - 4\alpha_e - \alpha_{e'} - \alpha_{e''})$$

$$- \frac{2EI_{ed}}{h}\left[(2\alpha_e + \alpha_d)\cos^2\theta_{ed} + (2\alpha_e + \alpha_f)\cos^2\theta_{ef}\right]$$

$$+ \frac{E_T I_T}{h}\left[(\alpha_d - \alpha_e)\sin^2\theta_{ed} + (\alpha_f - \alpha_e)\sin^2\theta_{ef}\right] = 0$$

Since h is constant, let $\frac{I_{ed}}{h} = q\frac{I_{ee'}}{L}$ and

$$\frac{E_T I_T}{2EI_{ed}} = m \quad \text{then} \quad \frac{E_T I_T}{h} = \frac{2mEI_{ed}}{h} = 2mq\frac{I_{ee'}E}{L}$$

(9) $$\therefore \quad \alpha_e\left[4+2q(\cos^2\theta_{ed}+\cos^2\theta_{ef})+qm(\sin^2\theta_{ef}+\sin^2\theta_{ed})\right]$$

$$+\alpha_d q(\cos^2\theta_{ed}-m\sin^2\theta_{ed})+\alpha_f q(\cos^2\theta_{ef}-m\sin^2\theta_{ef})=$$

$$= 3R_{e'}+3R_{e''}-\alpha_{e'}-\alpha_{e''}$$

In eq. (9) are 5 unknown quantities, α_e, $\alpha_{e'}$, $\alpha_{e''}$, α_d and α_f. All joints of one transverse frame will rotate in the same direction, hence α_d may be assumed $= \alpha_e = \alpha_f$ in eq. (9) with small resulting error. Since $\alpha_{e'}$ and $\alpha_{e''}$ depend on secondary stresses in next frame space it is not possible to solve for them directly, so it is necessary to assume that longitudinals at E' and E" remain perpendicular to frames E'F'D' and E"F"D" respectively.

$\therefore \alpha_{e'} = g_{e'}$ (see p.1).

Since counterclock rotation of frame E"F"D"($+\alpha_{e''}$) occurs when ΔL_o is negative, $\alpha_{e''} = -g_{e''}$.

Substituting in (9)

(10) $$\alpha_e = \frac{3R_{e''} + 3R_{e'} - g_{e'} + g_{e''}}{4 + 3q(\cos^2\theta_{ed} + \cos^2\theta_{ef})}.$$

In eq. (10) the torsional effect has disappeared since α_d was assumed equal to α_e and α_f.

By change of notation in eq. (10),

$$\alpha_d = \frac{3R_{e''} + 3R_{e'} - g_{e'} + g_{e''}}{4 + 3q(\cos^2\theta_{ed} + \cos^2\theta_{dc})}.$$

which does not agree with assumption that $\alpha_d = \alpha_e$ by an amount $(\cos^2\theta_{dc} - \cos^2\theta_{ef})3q$ in denominator. However, the resulting error in value of α_e is small (see Appendix 2).

Substituting value of α_e and $\alpha_{e'}$ in eq. (5),

$$M_{ee'} = \frac{2EI}{L}\left(3R_{e'} - 2\,\frac{3R_{e'} + 3R_{e''} - g_{e'} + g_{e''}}{4 + 3q(\cos^2\theta_{ed} + \cos^2\theta_{ef})} - g_{e'}\right)$$

Let $q(\cos^2\theta_{ed} + \cos^2\theta_{ef}) = Z_e$ $\quad q(\cos^2\theta_{ed} + \cos^2\theta_{dc}) = Z_d$ etc.

(11) $$\therefore\ M_{ee'} = \frac{2EI}{L}\left[\frac{(2 + 3Z_e)\ (3R_{e'} - g_{e'}) - 6R_{e''} - 2g_{e''}}{4 + 3Z_e}\right]$$

Substituting value of α_e and $\alpha_{e''}$ in eq. (7)

(12) $$M_{ee''} = \frac{2EI}{L}\left[\frac{(2+3Z_e)\ (3R_{e''}+g_{e''}) - 6R_{e'} + 2g_{e'}}{4 + 3Z_e}\right]$$

$$\Delta_{ee'} = \Delta y_{f'} + \Delta y_{m'} \quad \text{for left panel} \quad (\text{see p. 1})$$

$$\Delta_{ee''} = \Delta y_{f''} - \Delta y_{m''} \quad \text{since + moment produces - defl.}$$

$$\Delta y_f = \frac{1}{A_d}\left[\frac{FL}{E_d} - g\left(J + \frac{1}{2}LA_d\right)\right] \qquad (\text{p. 8})$$

$$J = J' - \frac{1}{2}LA_d \qquad (\text{p. 8})$$

$$\therefore \ \Delta_{ee'} = \frac{1}{A'_d}\left[\frac{F'L}{E_d} - g_{e'}J'_{'}\right] + \frac{1}{2}g_{e'}L$$

$$\frac{\Delta_{ee'}}{L} = R_{e'} = \frac{F'}{A'_dE_d} + g_{e'}\left(\frac{1}{2} - \frac{J'_{'}}{LA'_d}\right)$$

$$\frac{\Delta_{ee''}}{L} = R_{e''} = \frac{F''}{A'_d} - g_{e''}\left(\frac{1}{2} + \frac{J'_{''}}{LA''_d}\right)$$

$$g_{e'} = \frac{s'L}{E_Ly_L} \qquad g_{e''} = \frac{s''L}{E_Ly_L}$$

Substituting in (11)

$$M_{ee'} = \frac{2EI_{ee'}}{L(4+3Z_e)}\left\{[2+3Z_e]\left[\frac{3F'}{A'_dE_d} + 3g_{e'}\left(\frac{1}{2} - \frac{J'_{'}}{LA'_d}\right) - g_{e'}\right] - \frac{6F''}{A''_dE_d} + \right.$$

$$\left. + 6g_{e''}\left(\frac{1}{2} + \frac{J'_{''}}{LA''_d}\right) - 2g_{e''}\right\}$$

$$(13) \quad M_{ee'} = \frac{2I_{ee'}}{4+3Z_e}\left\{3Ze\left[\frac{3F'}{nA'_dL} + \frac{s'}{2y_{Lm}}\left(1 - \frac{6J'_{'}}{LA'_d}\right)\right] + \frac{6}{nL}\left(\frac{F'}{A'_d} - \frac{F''}{A''_d}\right)\right.$$

$$\left. + \frac{s'+s''}{y_{Lm}} - \frac{6}{Ly_{Lm}}\left(\frac{s'J'_{'}}{A'_d} - \frac{s''J'_{''}}{A''_d}\right)\right\}$$

When all wires act $J'_{'} = J'_{''} = 0$, $A'_d = A''_d$ and eq. (13) becomes

$$(14) \quad M_{ee'} = \frac{2I_{ee'}}{4+3Z_e}\left\{3Z_e\left[\frac{3F'}{nA_dL} + \frac{s'}{2y_L}\right] + \frac{6}{nL}\left[\frac{F'-F''}{A_d}\right] + \frac{s'+s''}{y_L}\right\}$$

By making similar substitutions in (12) and collecting,

$$(15)\quad M_{ee''} = \frac{2I_{ee'}}{4+3Z}\left\{3Z_e\left[\frac{3F''}{nA''_dL} - \frac{s''}{2y_L}\left(1 + \frac{6J'_{''}}{LA''_d}\right)\right] + \frac{6}{nL}\left(\frac{F''}{A''_d} - \frac{F'}{A'_d}\right)\right.$$

$$\left. - \frac{s'+s''}{y_L} - \frac{6}{Ly_L}\left(\frac{s''J'_{''}}{A''_d} - \frac{s'J'_{'}}{A'_d}\right)\right\}$$

When all wires act,

$$(16)\quad M_{ee''} = \frac{2I_{ee''}}{4+3I_e}\left\{3Z_e\left[\frac{3F''}{nA_dL} - \frac{s''}{2y_L}\right] + \frac{6}{nL}\left[\frac{F''-F'}{A_d}\right] - \frac{s''+s'}{y_L}\right\}$$

When $F' = F'' = 0$, $s' = s''$ since $M'_{'} = M'_{''}$, and equa. (14)&(15)

reduce to $M_{ee'} = I_{ee'} = \frac{s'}{y_L}$ $\qquad M_{ee''} = -I_{ee''}\frac{s''}{y_L}$

When $F'' = -F'$ and $M = 0$

$$M_{ee'} = 6I_{ee''}\frac{F''}{nA_dL} \qquad M_{ee''} = -6I_{ee'}\frac{F'}{nA_dL}$$

From eq. (4) – $\quad P_{ee'} = \frac{3M_{ee'}}{L} - \frac{6EI_{ee'}}{L^2}(R_{e'} - \alpha_e)$

Substituting value of $M_{ee'}$ from eq. (11) and α_e from (10)

$$P_{ee'} = \frac{6EI_{ed}}{L^2}\left[\frac{(2+3Z_e)\ (3R_{e'}-g_{e'}) - 6R_{e''} - 2g_{e''}}{4+3Z_e} - R_{e'} + \right.$$

$$\left. + \frac{3R_{e'} + 3R_{e''} - g_{e'} + g_{e''}}{4+3Z_e}\right]$$

Substituting values of R_e, and g_e, (See p. 32)

$$(17)\quad P_{ee'} = \frac{6I_{ee'}}{L(4+3Z_e)}\left\{\frac{6Z_e+5}{LA'_d}\left[\frac{F'}{n} - \frac{s'J'_{'}}{y_L}\right] - \frac{s'-s''}{2y_L} - \frac{3}{LA''_d}\left[\frac{F''}{n} - \frac{s''J'_{''}}{y_L}\right]\right\}$$

When all wires act,

$$(18)\quad P_{ee''} = \frac{6I_{ee'}}{L(4+3Z_e)}\left[\frac{(6Z_e+5)F'}{LA_dn} - \frac{3F''}{LA_dn} - \frac{s'-s''}{2y_L}\right]$$

Similarly,

$$(19) \quad P_{ee''} = \frac{6I_{ee''}}{L(4+3Z_e)}\left\{\frac{6Z_e+5}{LA''_d}\left[\frac{F''}{n} - \frac{s''J''}{y_L}\right] + \frac{s''-s'}{2y_L} - \frac{3}{LA'_d}\left[\frac{F'}{n} - \frac{s'J'}{y_L}\right]\right\}$$

When all wires act

$$(20) \quad P_{ee''} = \frac{6I_{ee''}}{L(4+3Z_e)}\left\{\frac{(6Z_e+5)F''}{LA_dn} - \frac{3F'}{LA_dn} + \frac{s''-s'}{2y_L}\right\}$$

When F' and $F'' = 0$, $\quad P_{ee'} = 0$ and $P_{ee''} = 0$

When $M' = 0$ and $F'' = -F'$

$$P_{ee'}L = \frac{12I_{ee'}F'}{nA_dL} = 2 \times M_{ee'}$$

$$P_{ee''}L = -\frac{12I_{ee''}F''}{nA_dL} = 2 \times M_{ee''}$$

Special cases:

When $F' = F'' = 0$ $\quad M_{ee'} = I_{ee'}\frac{s'}{y_L}$ and $P_{ee'} = 0$

Let e = distance from neutral axis of longitudinal to any fiber and s_2 = secondary unit stress.

Then $s_2 = e\frac{s'}{y_L}$

Total unit stress on this fiber =

$$s' + s_2 = s' + e\frac{s'}{y_L} = \frac{s'}{y_L}(y_L+e)$$

This indicates that for this case the unit stress in any part of a longitudinal is proportional to the distance of that part from the neutral axis of the ship.

When $F'' = -F'$ and $M''_1 = M'_1 = 0$

$$M_{ee'} = 6I_{ee'}\frac{F'}{nA_dL}, \quad P_{ee'} = \frac{2M_{ee'}}{L}$$

These formulae check against the formula for the deflection of a fixed ended beam loaded in the center. Deflection of beam of span = 2L with load w at center =

$$\Delta y = \frac{w \times (2L)^3}{192EI} = \frac{wL^3}{24EI}$$

and moment of center $= \frac{w(L \times 2)}{8} = M_c = \frac{wL}{4}$

$$\therefore \quad \Delta y = \frac{M_c L^2}{6EI}$$

From eq. (V) p. 6 $\quad \Delta y = \frac{FL}{A_d E_d}$ when $M' = 0$

$$\therefore \quad \frac{M_c L^2}{6EI} = \frac{FL}{A_d E_d}$$

$$M_c \doteq \frac{6FI}{nA_d L} = M_{ee'}$$

$$w = \frac{4M_c}{L} = \frac{4M_{ee'}}{L} = 2\,P_{ee'}$$

Computations made hereinafter for the numerical value of secondary stresses show that the terms involving s and s' are very small relative to the shear terms. Therefore it will usually be sufficiently accurate to omit them. In Design Memo #16, Mr. Burgess has suggested that they be combined with the primary stresses, which is also satisfactory. The omission of these terms reduces the formulae to simpler form. If also we let c = distance from c. of g. of any channel of a longitudinal to a horizontal axis through the c. of g. of the member then the stress in the member will be $M_{ee'}c/I_{ee'}$ and if we substitute $D_e = \frac{1}{2}(4 + 3Z_e)$

then $s_{ee'} = \frac{6c}{nA_d L}\left(F' - \frac{(F'+F'')}{D_e}\right)$ = secondary unit stress in member EE'

$s_{ee''} = \frac{6c}{nA_d L}\left(F'' - \frac{(F''+F')}{D}\right)$ = secondary unit stress in member EE"

$P_{ee'} = \frac{5I_{ee'}}{nA_d L^2}\left(2F' - \frac{1.5(F'+F'')}{D_e}\right)$ = secondary shear in member EE'

$$P_{ee''} = \frac{6I_{ee''}}{nA_dL^2}\left(2F'' - \frac{1.5(F''+F')}{D_e}\right) = \text{secondary shear in member } EE''$$

Formulae follow similarly for secondary stresses in the tranverse members.

The term Z involves the two angles θ of the transverse members adjacent to it, and it should be kept clearly in mind that these members extend from main longitudinal to main longitudinal, and not to the intermediate ones. Let u be the angle made with the horizontal by the radius to the joint in question (say E), this angle being measured in the vertical plane. This radius approximately bisects the angle between the two transverse members at the joint (except at the M girder). Draw a tangent line at the joint, perpendicular to this radius, and let the angles between it and the transverse members be b. Then it is clear that $\theta_{ed} = u + b$, and that $\theta_{fe} = u - b$.

Then $Z_e = q(\cos^2\theta_{ed} + \cos^2\theta_{fe}) = q(\cos^2(u_e+b) + \cos^2(u_e-b))$

$$= q(1+\cos 2u_e \cos 2b)$$

and $\frac{1}{2}(4+3Z_e) = (2 + 1.5\,q) + 1.5\,q\cos 2b\cos 2u = D_e$

In this expression it is seen that q and b are constant for a ship of the customary design which has equal panels on 24 out of 25 sides, and that u is the only variable. Attention is called to the fact that when u reaches the value of 45°, and $2u = 90^\circ$, then $\cos 2u$ becomes negative, so that if b could be zero when $u = 90^\circ$, then D_e would reduce to 2, because the q terms would cancel. Such an assumption, however, is not warranted because $\cos 2b$ has a constant value of about 0.88. In studying the variation of the secondary stresses from longitudinal A to M, it should be noted that the distance c for apex channel varies as the sin u, but although this term diminishes, proceeding from A to M, the term D_e at first increases, so that the parenthesis as a whole increases and the variation of stress is therefore complex. Such a study shows, however, that the secondary stresses are maximum for C, D, E, and I, J, K, rather than at A or M or G.

Secondary stresses - Appendix I.

Degree of error involved in neglecting lateral moments. See Fig. 11.

Consider 4 stiff members as shown in sketch with transverses EA and AD inclined at angle θ to vertical. If D,E,C are fixed for rotation and deflection and B is fixed for rotation and deflects vertically a distance Δ,

component of Δ in plane DAB = $\Delta\cos\theta$
component of Δ perpendicular to plane DAB = $\Delta\sin\theta$

Let M_{ab} be bending moment in AB at A about axis perpendicular to DAB (i.e. radial axis)

M' = corresponding moment about tangential axis.

From eq. (2)

$$M_{ab} = \frac{2EI_{ab}}{L}\left(\frac{3\Delta\cos\theta}{L} - 2\alpha_a - 0\right)$$

$$M_{ac} = \frac{2EI_{ac}}{L}(0 - 2\alpha_a - 0)$$

$$M_{ad} = \frac{2EI_{ad}}{h}(0 - 2\alpha_a - 0)$$

$$M_{ae} = \frac{2EI_{ae}}{h}(0 - 2\alpha_a - 0)$$

For equilibrium $\Sigma M = 0$. Let $\frac{I_{ad}}{h} = \frac{I_{ae}}{h} = q\,\frac{I_{ab}}{L}$

then $$\frac{3\Delta\cos\theta}{L} - 4\alpha_a - 4q\,\alpha_a = 0$$

$$\alpha_a = \frac{3\Delta\cos\theta}{4L(1+q)}$$

$$\therefore\ M_{ab} = \frac{2EI_{ab}}{L}\left(\frac{3\Delta\cos\theta}{L} - \frac{3\Delta\cos\theta}{2L(1+q)}\right)$$

$$= \frac{3EI_{ab}\Delta\cos\theta}{L^2}\left(\frac{1+2q}{1+q}\right)$$

Taking the moments about tangential axis, neglecting torsion in AD and AE

$$M'_{ad} = 0 \qquad M'_{ae} = 0$$

$$M'_{ab} = \frac{2EI'_{ab}}{L}\left(\frac{3\Delta\sin\theta}{L} - 2\alpha'_a\right)$$

$$M'_{ac} = \frac{2EI'_{ac}}{L}(-2\alpha'_a)$$

$$\bar{\Sigma}M = 0 \qquad \therefore \quad \alpha'_a = \frac{3\Delta \sin\theta}{4L}$$

Let $\dfrac{I'_{ab}}{I_{ab}} = r,$ then $M'_{ab} = \dfrac{2ErI_{ab}}{L}\left(\dfrac{3\Delta\sin\theta}{L} - \dfrac{3\Delta\sin\theta}{2L}\right)$

$$= \frac{3ErI_{ab}\,\Delta\sin\theta}{L^2}$$

The resultant moment will be inclined to plane DAB by an angle γ, see Fig. 12, and

$$\tan\gamma = \frac{M'_{ab}}{M_{ab}} = \frac{3ErI_{ab}\,\Delta\sin\theta}{L^2} \div \frac{3EI_{ab}\,\Delta\cos\theta}{L^2}\,\frac{(1+2q)}{(1+q)}$$

$$= \tan\theta \times r\left(\frac{1+q}{1+2q}\right)$$

For the ZR-1,

$$r = \frac{I'_{ab}}{I_{ab}} = \frac{I \text{ for tang. axis}}{I \text{ for rad. axis}} = \frac{12.4}{5.05} = 2.46 \text{ for main longs.}$$

for main transverse frame I is constant for any axis.

$$I_{ae} = 7 \qquad h = 3\text{m.} \qquad L = 5\text{ m.} \qquad \therefore\ q = \frac{7}{5.05} \times \frac{5}{3} = 2.31$$

$$\therefore\ \tan\gamma = \tan\theta \times 2.46 \times \frac{3.31}{5.62} = 1.45\tan\theta$$

$$\tan(\gamma - \theta) = \frac{\tan\gamma - \tan\theta}{1 + \tan\gamma\tan\theta} = \frac{.45\tan\theta}{1 + 1.45\tan^2\theta}$$

To find maximum inclination of resultant moment to vertical plane $(\gamma - \theta)$, differentiate and equate to zero.

$$\frac{d\tan(\gamma-\theta)}{d\tan\theta} = \frac{.45(1 + 1.45\tan^2\theta) - .45\tan\theta(2.9\tan\theta)}{(1 + 1.45\tan^2\theta)^2} = 0$$

$\therefore$ for max. $\tan(\gamma - \theta)$, $\tan\theta = \sqrt{.69} = .83$ $\theta = 39^\circ 45'$

$$\therefore \quad \max.\ \tan(\gamma - \theta) = \frac{.45 \times .83}{1 + 1.45 \times .83^2} = .187$$

$$\therefore \quad (\gamma - \theta)\max = 10^\circ 30'$$

Since $\cos 10^\circ 30' = 0.983$, the maximum error in resultant moment due to neglecting lateral component is 1.7%.

Secondary Stresses - A pendix 2.

Error in value of α_e due to assumption that $\alpha_d = \alpha_e = \alpha_f$

Eq. (9) $\alpha_e\ [4+2q(\cos^2\theta_{ed} + \cos^2\theta_{ef}) + qm(\sin^2\theta_{ef} + \sin^2\theta_{ed})] +$

$$\alpha_d\ q(\cos^2\theta_{ed} - m\sin^2\theta_{ed}) + \alpha_f q(\cos^2\theta_{ef} - m\sin^2\theta_{ef}) =$$

$$= 3R_e' + 3R_e'' - \alpha_e' - \alpha_e''$$

Since α_e' and α_e'' were assumed $= g_e'$ and g_e'' respectively,

the right-hand term of this equation is a constant for any given condition of loading. Call this constant "C".
α_d was assumed $= \alpha_e$ $\therefore$ from eq. (10)

$$\alpha_d = \frac{C}{4+3q(\cos^2\theta_{ed} + \cos^2\theta_{ef})},$$

but since eq. (10) is a general equation for any joint, by change of notation,

$$\alpha_d = \frac{C}{4+3q(\cos^2\theta_{ed} + \cos^2\theta_{dc})}$$

Calling the first value α_{d1}, the second α_d, and the difference ϵ_d, then $\epsilon_d = \alpha_d - \alpha_{d1}$, and similarly $\epsilon_f = \alpha_f - \alpha_{f1}$

$\epsilon_e = \alpha_e - \alpha_{e1}$, where α_e is value corresponding to α_d and α_{e1} is value from eq. (10).

From eq. (9), by changing subscripts and subtracting α_e' from α_e,

$$(21)\quad \epsilon_e\,[4+2q(\cos^2\theta_{ed}+\cos^2\theta_{ef})+qm(\sin^2\theta_{ed}+\sin^2\theta_{ef})]$$

$$+\,\epsilon_d\, q(\cos^2\theta_{ed}-m\sin^2\theta_{ed})+\epsilon_f q(\cos^2\theta_{ef}-m\sin^2\theta_{ef})=0$$

Now $$\frac{\epsilon_d}{\alpha_{e'}}=\frac{\alpha_d-\alpha_{d'}}{\alpha_{e''}}=\frac{3q(\cos^2\theta_{ef}-\cos^2\theta_{dc})}{4+3q(\cos^2\theta_{ed}+\cos^2\theta_{dc})}$$

and $$\frac{\epsilon_f}{\alpha_{c'}}=\frac{3q(\cos^2\theta_{ed}-\cos^2\theta_{fq})}{4+3q(\cos^2\theta_{ef}+\cos^2\theta_{fq})}$$ by changing subscripts.

Let $\cos^2\theta_{dc}=u$, $\cos^2\theta_{de}=v$, $\cos^2\theta_{ef}=w$ and $\cos^2\theta_{fq}=z$

$$2q\cos^2\theta+qm\sin^2\theta=q\,[(2-m)\cos^2\theta+m]$$

$$\cos^2\theta-m\sin^2\theta=\cos^2\theta\,(1+m)-m$$

Dividing eq. (21) by α_{e1} and substituting above values,

$$\frac{\epsilon_e}{\alpha_{e1}}\,[4+q(2-m)\,(v+w)+2qm\,]=3q^2\left\{\frac{(u-w)\,[v(1+m)-m]}{4+3q(u+v)}+\frac{(z-v)\,[w(1+m)-m]}{4+3q(w+z)}\right\}$$

For the ZR-1 $q = 2.31$ (see p. 38) and since $I_T = 2I_{ed}$, $m=\frac{E_T}{E}$ which we will assume $=\frac{1}{2}$.

$$\frac{\epsilon_e}{\alpha_{e1}}=\frac{8\left[\frac{(u-w)\;(3v-1)}{4+7\;(u+v)}+\frac{(z-v)\;(3w-1)}{4+7\;(w+z)}\right]}{6.31+3.47(v+w)}\,.$$

Values of the ratio $\frac{\epsilon_e}{\alpha_{e1}}$ expressed in % have been plotted in Fig. 13 for one quadrant of regular 24 sided ship. This curve sho maximum discrepancy to be 6% and occurs for top longitudinal. As this is merely the discrepancy between value of α_e from eq. (10) and value found by change of notation, the true value of α for any joint will differ from value computed from eq. (10) by less than the plotted discrepancy.

Computation of Combined Primary and Secondary Stresses in Frame Space 80 - 90.

Case (a) - Maximum tensile stress, "C" girder, Bow 6° down.

Frame number	#80		#90		#100
shear: Dynamic (Memo. #6, Table IV)		- 530		-1059	
Static (" #6, " VIa)		-2190		+3885	
Moment: Dynamic	222080	-2720	216780	+2826	206190 (hog'g)
Static	160530		138630		177460 (hog'g)
	382610		355410		383650 (hog'g)
Moment at center of frame space=M'		369010		369530	
Unit bending stress (M'×0.0199) (See Memo. #7, Table 3)		+7350#/s.in.		+7260#/s.in.	

Secondary bending moment in member 80-90 at joint 90 =(eq. (14),p. 32)

$$M_{cc'} = \frac{2I_{cc'}}{4+3Z_c}\left(3Z_c\left(\frac{3F'}{nA_dL} + \frac{s'}{2y_L}\right) + \frac{6}{nA_dL}(F' - F'') + \frac{s'+s''}{y_L}\right)$$

$Z_c = q(\cos^2\theta_{bc} + \cos^2\theta_{cd}) = 2.31(\cos^2 75.5^{\circ} + \cos^2 46.5^{\circ})$

$= 2.31(0.25^2 + 0.688^2) = 1.24 \qquad 3Z_c = 3.72$

$n = E_d/E_L = 2.86 \qquad A_d = 0.057$ (See Table 5, Memo. #7)

$L = 10$ met. $= 394'' \quad nA_dL = 2.86 \times 0.057 \times 394'' = 64.2''^3$

$F' = -2720 \quad F'' = +2826 \quad s' = +7350 \quad s'' = +7260 \quad y_L = 410''$

$$M_{cc'} = \frac{2I_{cc'}}{4+3.72}\left((3.72)\left(\frac{3(-2720)}{64.2} + \frac{7350}{2\times 410}\right) + \frac{6}{64.2}(-2720 - 2826) + \frac{7350 + 7260}{410}\right) = -2411_{cc'} \text{ in lbs.}$$

The negative sign indicates a clockwise moment, i.e. tension on the lower channels and compression on the apex channel.

For girder "C" the distance from the horizontal neutral axis to c. of g. of lower base channel is about 7", hence the section modulus will be $I_{cc'}/7$, therefore, unit stress in base channel $= 241\ I_{cc'}/I_{cc'}/7 = 241 \times 7$

$= 1687$ lbs. per sq. in. tension.

also, unit stress in apex channel $= 241 \times 7.5 =$

$= 1800$ lbs. per sq. in. compr.

Therefore, the maximum combined stress for this case will be:
(See Des. Memo. #7, Table 9.)

Initial stress	-1280	lbs. per sq. in.
End load s'	+7350	lbs. per sq. in.
Gas pressure bending str.	+6697	lbs. per sq. in.
Secondary stress	+1690	lbs. per sq. in.
Total combined stress	+14459	lbs. per sq. in.

Case (b) - Maximum compressive stress, "A" girder, Bow 6° up.

Frame number	#80		#90		#100
Shear: Dynamic (Memo. #6, Table IV)		+ 530#		+1059#	
Static (" #6, " VI)		-4624#		+1449#	
		-4094#		+2508#	
Moment: Dynamic	-222080		-216780		-206190 (sag'g)
Static	+ 57060		+ 10820		+ 25310 (hog'g)
	-165020		-205960		-180880 (hog'g)
Moment at center of frame space=M'		-185490		-193420	
Unit bending stress (M' x 0.0224) (See Memo. #7, Table 3.)		- 4150#/s.in.		- 4330#/s.in.	

$Z_a = 2.31(0.25^2 + 0.25^2) = 0.29$, $3Z_a = 0.87$, $y_L = 467''$ (Memo. #7, Table 1)

$F' = -4094$, $F'' = +2508$, $s' = -4150$, $s'' = -4330$

$$M_{aa'} = \frac{2I_{aa''}}{4+0.87}\left(0.87\left(\frac{3 \times -4094}{64.2} + \frac{-4150}{2 \times 467}\right) + \frac{6}{64.2}(-4094 - 2508)\right.$$

$$\left. + \frac{-4150 - 4330}{467}\right) = \frac{2 \times 804}{4.87} I_{aa'} = -332 I_{aa'}$$

The negative sign indicates a clockwise moment, i.e. a compression on the apex channel.

For girder "A" the distance from the (horizontal) neutral axis to c. of g. of apex channel is about 8.3" hence the section modulus will be $I_{aa'}/8.3$,

therefore, unit stress in apex channel = 332 × 8.3 (as before)

= 2760 lbs. per sq. in. compr.

Therefore, the maximum combined stress for this case will be:
(See Des. Memo. #7, Table 9).

Initial stress	-1175	lbs. per sq. in.
End load s'	-4150	lbs. per sq. in.
Gas pressure bend. stress	-8305	lbs. per sq. in
Secondary stress	-2760	lbs. per sq. in.
Total combined stress	-16390	lbs. per sq. in. (compressed)

Comments: The theory as evolved for secondary stresses is based upon the deflection of one frame with reference to another, and the deflection used herein is that figured under primary stresses. In reality the secondary bending moments of the longitudinals help to resist this shearing deflection, and therefore the resultant deflection is less, i.e. the secondary stresses will be somewhat reduced (perhaps 5 or 10%). On the other hand, these computations neglect the rigidity of the secondary transverses (a panel length of 10 meters being used)} and are therefore somewhat too small on that account, because if these are considered the restraint is increased, the angle of curvature at the main transverses is increased, and therefore the bending stresses. However, these two effects will more or less balance, and it is probably sufficiently accurate to use the stresses set forth above. (If anything, the stresses are larger than those set forth, rather than smaller.)

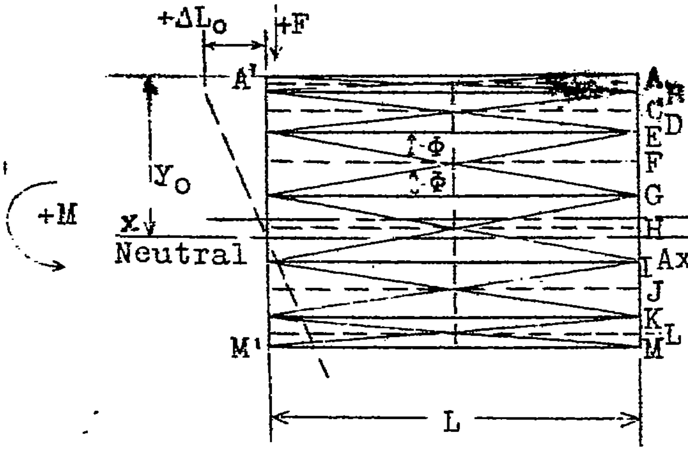

Fig.1a Elevation

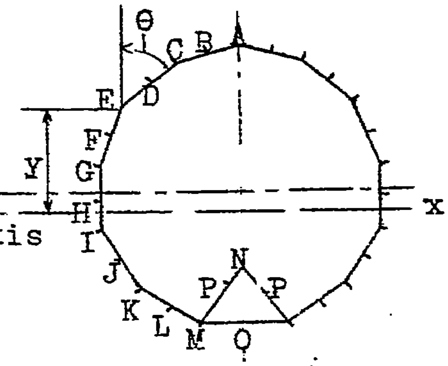

Fig.1b Section

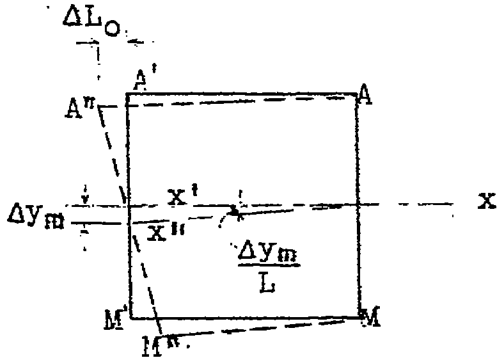

Fig.2 Deflection due to moment

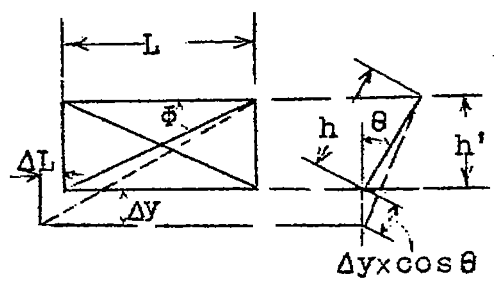

Fig.3

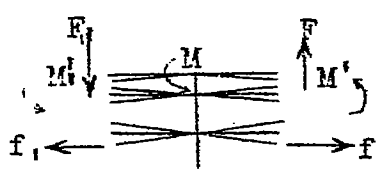

Fig.4

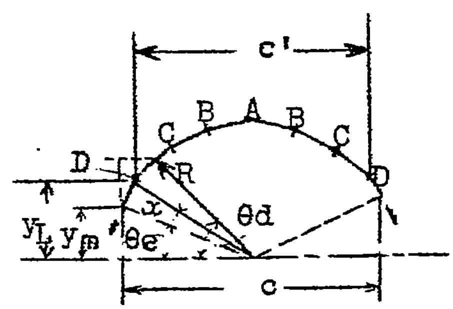

Fig.5

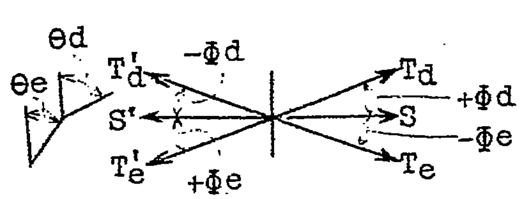

Fig.6

Plate I

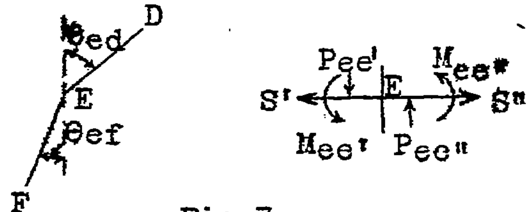

Fig.7.

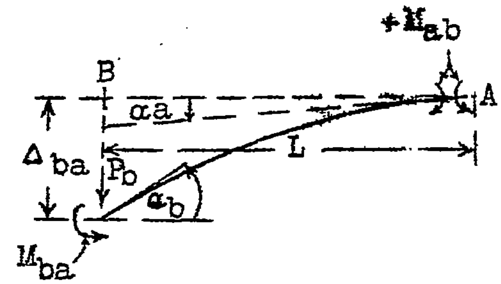

Fig.8.

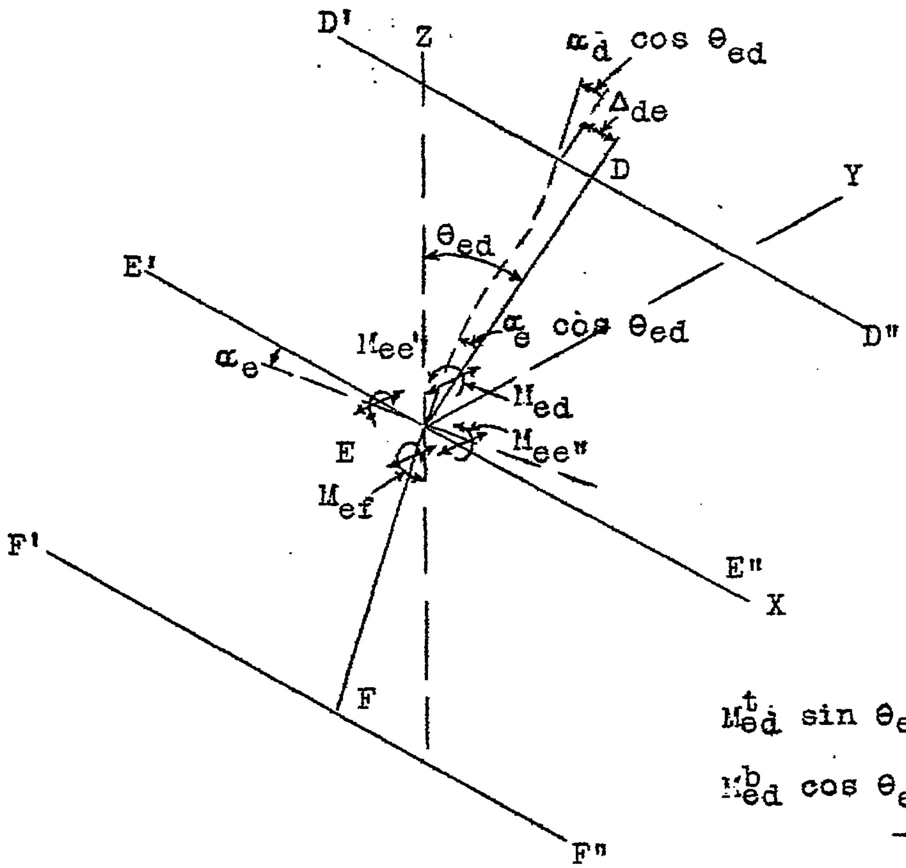

Fig.9.

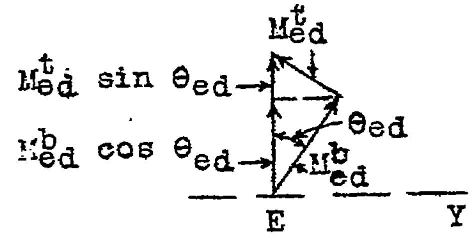

Fig.10.

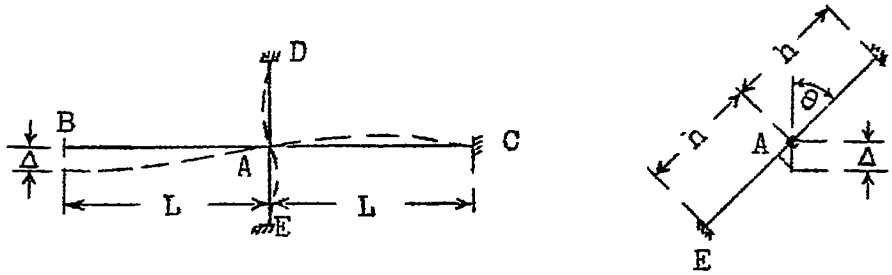

Fig.11.

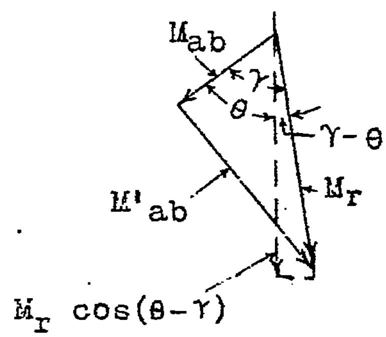

Fig.12.

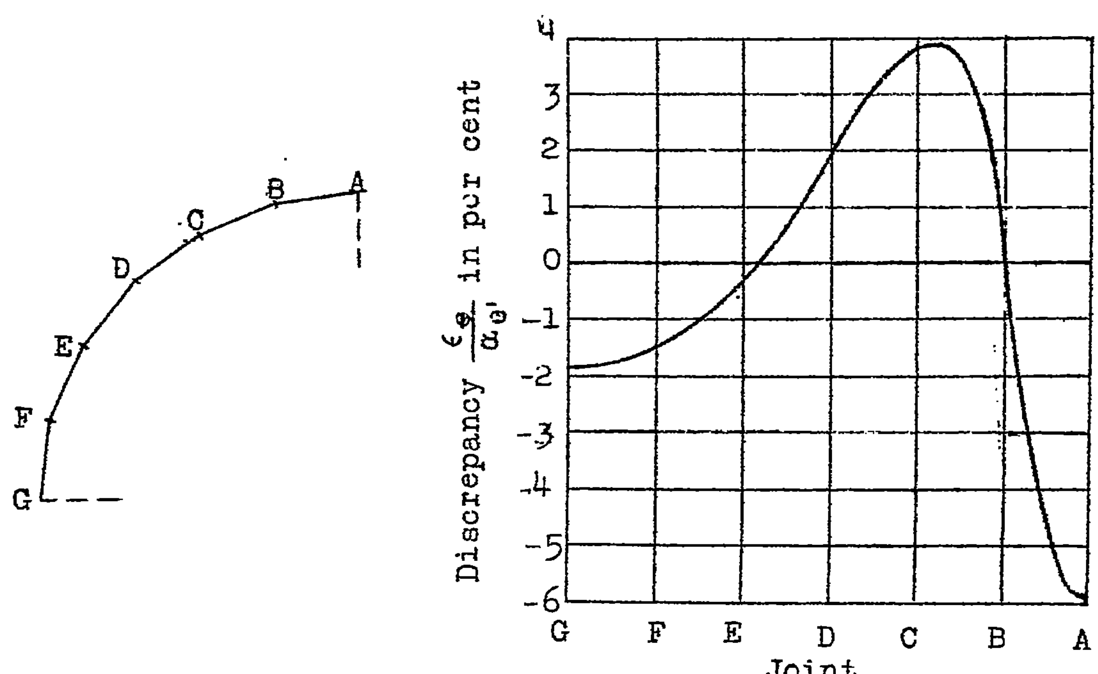

Fig.13.

Plate III.