Entropy Analysis of Microscopic Diffusion Phenomena in Diesel Sprays*

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Rapid mixing of fuel and air is an essential factor in improving combustion and emissions of diesel engines, and it is important to know the relationship between the microscopic structure of the heterogeneous distribution of fuel clouds and the local turbulence structure. This paper investigates local diffusion phenomena in sprays with focusing on scales of fuel cloud and eddies based on a newly developed entropy method. The results show that the diffusion intensity is the highest in the vicinity of the nozzle exit, and the heterogeneity scale is the smallest here. The heterogeneity scale increases gradually along the spray axis towards the downstream, with smaller size scales in the large clouds. In the downstream region, small-scale structures diffuse and become unclear, while large scale structures clearly remain. The paper details the microscopic structure of the heterogeneity in diesel sprays, and it demonstrates availability of the entropy method.

Key Words: Homogeneity, Heterogeneity, Diffusion, Scale, Entropy

1. Introduction

Rapid mixing of fuel and air is an essential factor in improving combustion and emissions of diesel engines. However, the relationship between the micro structure of the heterogeneous distribution of fuel clouds and the local turbulence structure is not well understood. Additionally there is no appropriate index for the analysis of the degree and size scale of heterogeneity. This paper investigates local diffusion phenomena focusing on the size scales of fuel clouds and eddies based on the newly developed entropy method.

In an application of this analysis, an investigation was made of the heterogeneous structure of liquid jets injected into water. Observation was also made of diesel sprays injected in high-pressure atmospheres. The results show that diffusion speed decreases and the size scale of the heterogeneous clouds increases from the vicinity of the nozzle toward top of the spray. The diffusion intensity is strong upstream of the jet, and it becomes weak in the downstream direction; this corresponds to strength and number density of the vorticity.

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There is much work on the structure of diesel sprays, however little concentrating on the microscopic diffusion process of fuel and air. Particularly, research into the size scale of fuel clouds and mixing is limited. Related to diffusion phenomena, Komori and Ueda defined an index of the mixing degree, which is derived from a statistical analysis of fluctuations in local concentrations [1][2]. This method requires the measurement of a number of fluctuation data over time and it gives information of the mixing degree for a single point. Mudford et al. and Bilger et al. investigated the validity of this concept and the range of the values of the mixing degree [3][4].

The heterogeneity degree can be expressed with the PDF (probability density function) of concentrations. Ikegami and Shioji reported a stochastic model using PDF to express the diffusion process of fuel and air [5]. Pope reviewed research of the probability density function on the transport equation in the combustion field [6]. Lemoine developed a method to measure velocity and concentration simultaneously at a single point, and investigated turbulent diffusion intensity downstream of grids [7].

All of these methods relate to expressing the degree of heterogeneity. However, neither of the methods is suited for an analysis of the diffusion state and size scale of a heterogeneity observed in a single picture of a spray.

2. Entropy Analysis

In statistical dynamics, entropy is a state quantity, which shows the probability of the state, and it increases with the progress of uniformity in the mixing process. In the research reported here it was attempted to extend this quantity to express the homogeneity degree of the state as observed in pictures and to develop a method applicable to research on diffusion processes in diesel sprays.

In statistical dynamics, entropy is defined as the logarithm of combination W in the distribution state of N particles, which gives the same state of spatial distribution of the particles. For example, a space is divided into M cells, and with a total number of particles N in each cell in the order $\{N_1, N_2, N_3 ... N_M\}$. Then the combination of particles, which gives the same number of distribution, becomes [8]:

$$W = {}_N C_{N_1} \times {}_{(N-N_1)} C_{N_2} \times \cdots \times {}_{N_M} C_{N_M} = \frac{N!}{N_1! N_2! N_3! \cdots N_M!} = \frac{N!}{\Pi N_i!} \quad \ldots (1)$$

As the entropy is the natural logarithm of the combination multiplied by a Boltzmann constant, it becomes the following equation, when Stirling approximation is applied for the case of N>>1 [8]:

$$S = k \ln(W) = k\left[N \cdot \ln(N) - \sum_i^M \{N_i \cdot \ln(N_i)\} \right] \ldots (2)$$

where k is a Boltzmann constant. Here it was assumed that the particle number N_i in each cell in the above equation could be linearly correlated to the local luminance $I(i)$ of an image:

$$S = \alpha \cdot \left\{ \sum_i^M I_{(i)} \right\} \cdot \ln\left\{ \alpha \cdot \sum_i^M I_{(i)} \right\} - \sum_i^M \left[\alpha \cdot I_{(i)} \cdot \ln\{\alpha \cdot I_{(i)}\} \right]$$
$$= \alpha \cdot \left\{ \sum_i^M I_{(i)} \right\} \cdot \ln\left\{ \sum_i^M I_{(i)} \right\} - \alpha \cdot \sum_i^M \left[I_{(i)} \cdot \ln\{I_{(i)}\} \right] \quad \ldots (3)$$

where α is a constant including the Boltzmann constant and the transformation coefficient of luminance to the particle number. The $I(i)$ is the luminance value in cell number i. In the definition of entropy, the number of cells is supposed to be infinitely large, and the size of the cells was set to the pixel size, as this is the smallest unit in the analysis of a picture.

The entropy value was normalized by the value for an uniform state with the same total number of particles, i.e. the same value as the integrated luminance of the picture. The luminance for the uniform picture is:

$$\overline{I_{(i)}} = \frac{1}{M} \cdot \sum_i^M I_{(i)} = \frac{I_t}{M} \quad \ldots (4)$$

where M is the number of cells in the space, and the I_t is an integrated value of the luminance over the whole space. The entropy of this homogeneous picture is then:

$$S_1 = \alpha \cdot I_t \cdot \ln(M) \quad \ldots (5)$$

Using the S_1, the normalized entropy S^* is defined as

$$S^* = \frac{S}{S_1} = \frac{I_t \cdot \ln(I_t) - \sum_i^M \{I_{(i)} \cdot \ln(I_{(i)})\}}{I_t \cdot \ln(M)} \quad \ldots (6)$$

The value is calculated directly from the luminosity distribution in pictures.

3. Validity of the Entropy Analysis

Table 1 is a list of the results of the entropy analysis for Figs. 1 to 3. Figure 1 is a comparison of tracers in an ethylene flame and a nitrogen jet. Figure 2 compares circles of different sizes and different clarities at the edge of the circles. Figure 3 are pictures artificially made to compare diffusion process of condensed particles, all with pixel 256 x 256. Comparing pictures in the each set, the ones with the most advanced state of diffusion has a larger entropy value. For example, tracers in the N_2 jet distribute more uniformly than the one in the C_2H_4 flame, giving the N_2 jet picture the larger entropy value. Similarly, as diffusion progresses from Phase 1 to Phase 4 in Fig. 3, the entropy value increases.

For Fig. 2, it could be expected that the diffusion in Case 2 is more progressed than in Case 1, because Case 2 has a group of objects broken into small clouds with the boundaries of the circles not sharp as in Case 1. However the entropy value of Case 2 is smaller than that of Case 1. Entropy becomes zero when all the particles composing the objects concentrate in a point. Here, Case 1 has a larger circle and this means that diffusion has progressed to a substantial extent, whereas in Case 2 the particles locate close to the center of each cloud and diffusion is not advance yet. It must be noted here that the brightness of the picture does not change the entropy value as can be recognized from Eq. (6), where attenuation of luminosity is present in both nominator and denominator, and so cancels out. Similarly Phase 4 in Fig. 3 could be expected to have larger en-

Table 1 Entropy values for the tested pictures

Pictures	Figure No.	Entropy S^*
C_2H_4	1	0.981
N_2	1	0.996
Case1	2	0.847
Case2	2	0.809
Phase1	3	0.813
Phase2	3	0.887
Phase3	3	0.932
Phase4	3	0.964

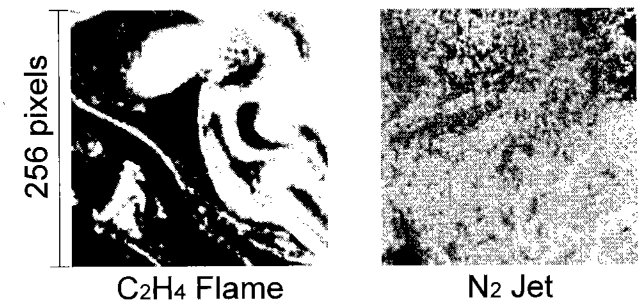

Fig. 1 Pictures of different tracer distributions

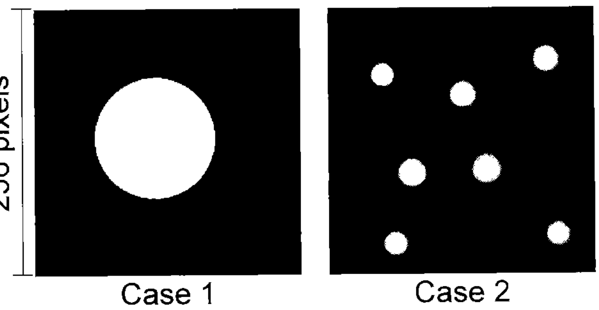

Fig. 2 Pictures with different size, number, and sharpness of circles

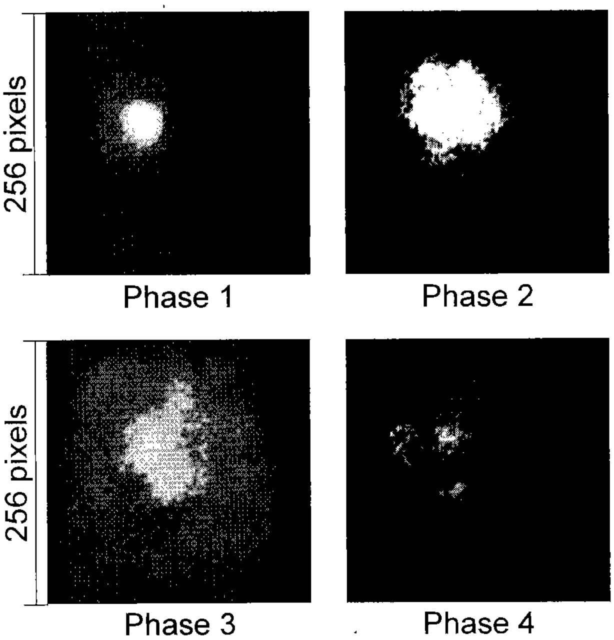

Fig. 3 Pictures expressing different states of diffusion

tropy than the C_2H_4 picture in Fig. 1. However when the integrated luminance I_t is set identical in the two figures, the Phase 4 picture has highly concentrated portion in the bright region. This appears to be the cause of lower entropy value in the Phase 4 picture.

Thus it is demonstrated that the homogeneity degree of the field can be expressed quantitatively by the entropy analysis proposed in this research. The values are mostly over 0.9 and the differences in the values are small, but the small differences in the numbers show apparent differences in the diffusion states. It may be possible to define the entropy to give larger differences in the value, but the above definition is free from the brightness of the picture and suited to evaluate the diffusion coefficient as mentioned in the literature [9].

It is also possible to express the state of the homogeneity with the PDF of the luminance or the concentration itself. The advantage of the entropy is that it expresses the degree of homogeneity with a single scalar quantity. Also, the PDF does not always correspond to the entropy as seen in Fig. 2, where Case 1 would gives complete separation of black and white in PDF, indicating a less diffusive state than Case 2, the opposite of the entropy analysis.

4. Analysis Method of the Heterogeneity Size Scale

It appears to be important to identify the size scale of the heterogeneity, because the diffusion intensity is related to relative scale of fluid cloud and eddies. For example a large eddy may simply rotate a small cloud without apparent diffusion, if the eddy is large relative to the cloud. At the same time small eddies will contribute only little to the diffusion of large fluid clouds. In this section, a method for determining the scale of extent of the heterogeneity is discussed, using the entropy analysis method.

When the entropy of an area is analyzed by scanning over a picture containing a white object as illustrated in the bottom left corner in Fig. 4, entropy varies as shown in Fig. 4. It has a peak value when the area overlaps the object. This is because the region appears more uniform when the object is in the window than when only the part of the object is in the window. When the analysis area is larger or smaller than the object, the entropy change in the vicinity of the peak is smaller than when the area size and the object are of similar sizes. Therefore, it may be possible to specify the size scale of the cloud from the entropy change obtained with various area sizes.

Figure 4 shows the result of a trial to confirm the above discussion. It shows the entropy change in scanning the image using window areas of various sizes. The object in this case has a Gaussian distribution in luminosity with a deviation scale of x_ρ. Here with the whole area for analysis of size x_0 was named the "window", and the area for scanning for entropy of size x_w as the "sub-window". The deviation size scale of the Gaussian distribution is 0.1 times for the window size, x_0. The sub-window size is expressed relative to the window size, x_w/x_0. With any sub-window the entropy decreases once as it approaches the center of the cloud, and then increases to the maximum at the center of the Gaussian distribution. The curvature in the entropy change is the largest with a window size of 0.3 in the figure, and smaller or larger sub-windows give smaller curvatures. Based on this, the following parameter is introduced and it is defined as the "degree of scale-correspondence, ρ":

$$\rho = -\left.\frac{\partial^2 S*}{\partial (x/x_w)^2}\right|_{S*\ peak} = -\left(\frac{x_w}{x_0}\right)^2 \cdot \left.\frac{\partial^2 S*}{\partial x*^2}\right|_{S*\ peak} \quad \ldots(7)$$

where x_0 is the size of the window and x_w is the sub-window size. The dimensionless length, $x*$, is normalized by x_0.

Figure 5 shows the degree of scale-correspondence for different profiles and size scales of objects: three different size scales of Gaussian distribution with deviation x_σ, and two square black-and-white objects with side lengths x_L. The abscissa is the dimensionless sub-window size, and the ordinate is the "degree of scale-correspondence". The sub-window size that gives the maximum "degree of scale-correspondence" appears at the points where the sub-window size is identical to the squares and at the points where the ratio of the standard deviation and sub-window size is 4.3 for the Gaussian distribution: in the later case, the size is approximately two times the region covered by the standard deviation, and it correspond to the tail of the Gaussian distribution. These results indicate the possibility of identifying the scale of a cloud with the degree of scale-correspondence.

The degree of scale-correspondence should be calculated on the center line of the objects. To identify the position of the object, a window is scanned in the x direction first, and at the peak position of the scale-correspondence it is then scanned in the y direction, establishing the center of the cloud. This is continued over the whole space and the average value of the degree of scale-correspondence is derived for this sub-window size. The locations of the clouds are also stored in the dataset. It is possible to have number of scale-correspondence values for different sub-window sizes for a single object. Among these numbers, the largest value of the scale-correspondence is chosen for the object, and the sub-window is set the size of the object.

In the left picture in Fig. 6 white squares are distributed in the space, and the square size is 0.06 relative to the window size. The right figure in Fig. 6 shows the result of the analysis, showing sub-window sizes and the locations of the objects with the method mentioned above. The original location and size of the objects are reproduced exactly. Clusters of objects are also analyzed with the sub-window size of 0.22. A black window of size 0.24 corresponds to the region sur-

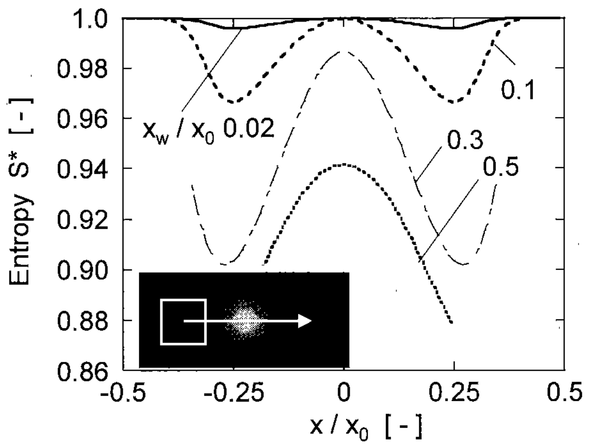

Fig. 4 Entropy Change in the process of scanning a window over a Gaussian concentration (x_σ/x_0=0.1)

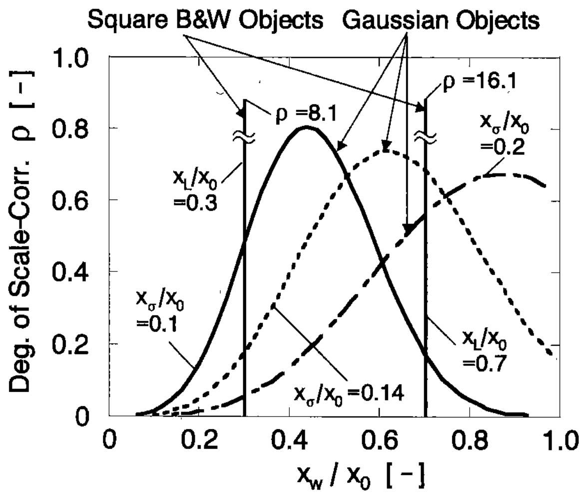

Fig. 5 Degree of scale-correspondence, ρ, for different sub-window size, x_w, for a variety size scales of Gaussian and square-black-and-white profiles

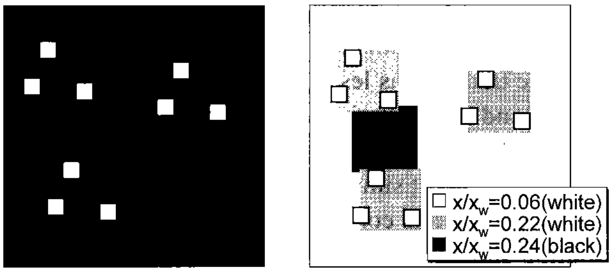

Fig. 6 Image applied to examine the number and size scales of objects (left), and the analyzed size scales and locations of objects (right)

rounded by the white objects, and it is recognized as a black heterogeneous-object. The number of clouds is counted with this method. For a thin, long object, the cloud number is one, and the size with larger scale-correspondence is chosen.

5. Analysis of Phenomena in Jets

To investigate the structure and process of diffusion phenomena in diesel sprays, a water jet was injected from a 0.6mm diameter nozzle into water to observe a process similar to a diesel spray at slower speeds. The injected water contained a fluorescent substance, and a cross section of the jet was visualized by an Ar laser sheet. Pictures were taken by a high-speed video camera at a speed of 4,500 frames per second. Figure 7 is an example of a picture, and the three locations marked on the jet axis were analyzed with the entropy method. Figure 8 shows pictures taken with a magnifying optical-setup at the upstream, midstream, and downstream positions of the jet indicated in Fig. 7 at 145.8ms after the start of injection. The size of the analyzed window is 2.8mm square.

Figure 9 is the results of the analysis, showing the size scales, entropy, and number of clouds. The number of clouds is the total number of clouds corresponding to each size scales. The entropy was calculated at the center of the cloud with a sub-window double the size of the one used for the scale-correspondence calculations. This value indicates the clearness of the cloud from the view including the neighbors, more distinctive with smaller entropy numbers, in other words more heterogeneous. The scale of cloud is expressed by dimensionless sub-window size with the maximum scale correspondence relative to the window size in Fig. 8 (2.8mm). At the upstream position, most of the heterogeneity is small scale, and there are few large clouds. In the midstream position, there are large heterogeneities with clear boundaries and this is indicated by the smaller entropy values. At the downstream position, the small size heterogeneities become unclear and the number decreases. Large size heterogeneities developed, and the boundaries are clearer than at the midstream position. These results show that the scale of heterogeneity increases and small size structures become indistinct as the observation point moves downstream.

It may be possible to measure diffusion intensities from the entropy change when tracing the sub-window in a Lagrangian manner. Figure 10 shows the distribution of the entropy change pattern calculated from the three consecutive pictures. A sub-window is traced in a Lagrangian manner among the three pictures with

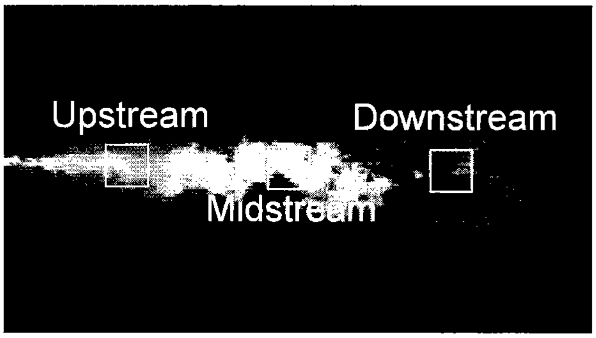

Fig. 7 Laser sheet picture of a Jet injected into water

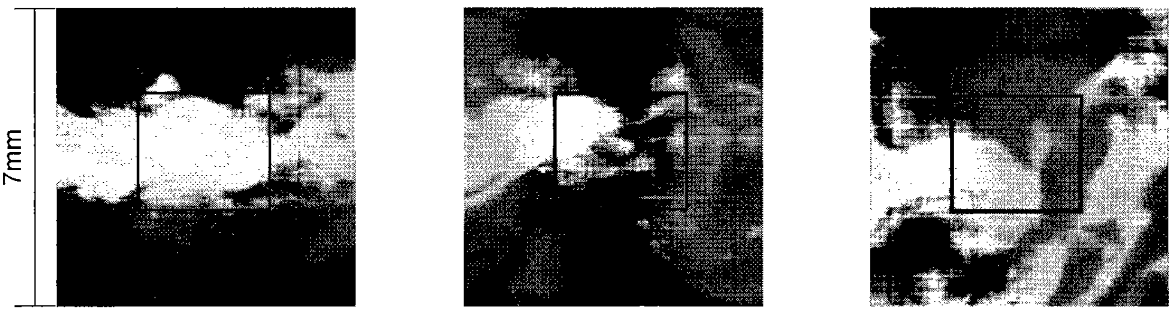

Fig. 8 Enlarged pictures for the analysis of heterogeneity degree and size scales

the PIV method with spatial correlation. The window size for the PIV analysis was set to be identical to the size used in the entropy analysis. Entropy was calculated and compared for each corresponding window, only the data which successfully traced over the three pictures were used for the analysis. The symbols in the figure indicate changes in entropy values among the three pictures. Pattern 1 indicates that the entropy increases continuously in the three pictures, and pattern 2 indicates that the entropy increases first and then decreases in the next step. The data number in the upstream region is smaller than at the other locations, because the tracing was less successful due to the higher velocity and lower clarity in the microscopic view before mixing.

It is seen that similar patterns gather together and their distribution is corresponding to the vorticity distribution shown in Fig. 11. For the vorticity analysis the size of the window was set to 0.33 relative to the 2.8mm picture size. This organized structure in the

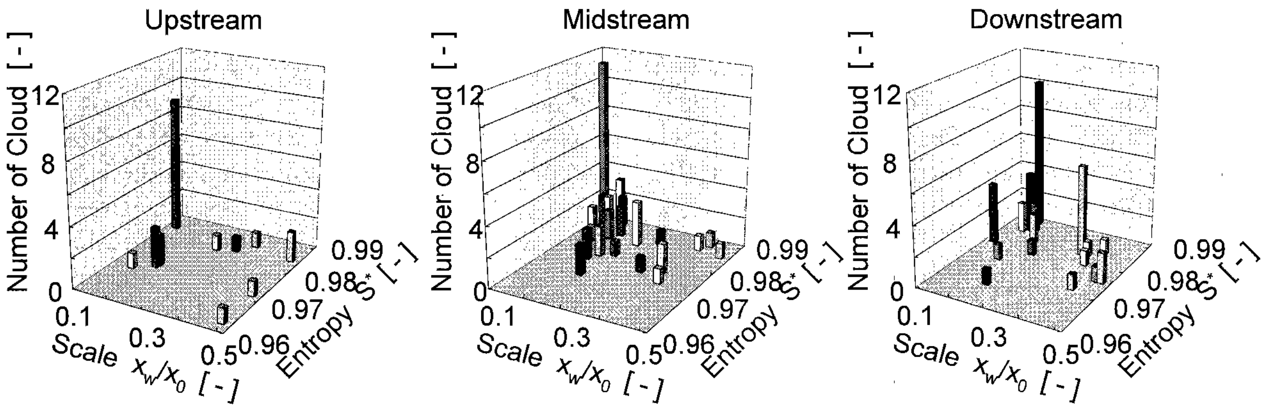

Fig. 9 Numbers, size scales and clearness of heterogeneous clouds: smaller entropy values indicate clearer boundaries of the cloud with less diffusion

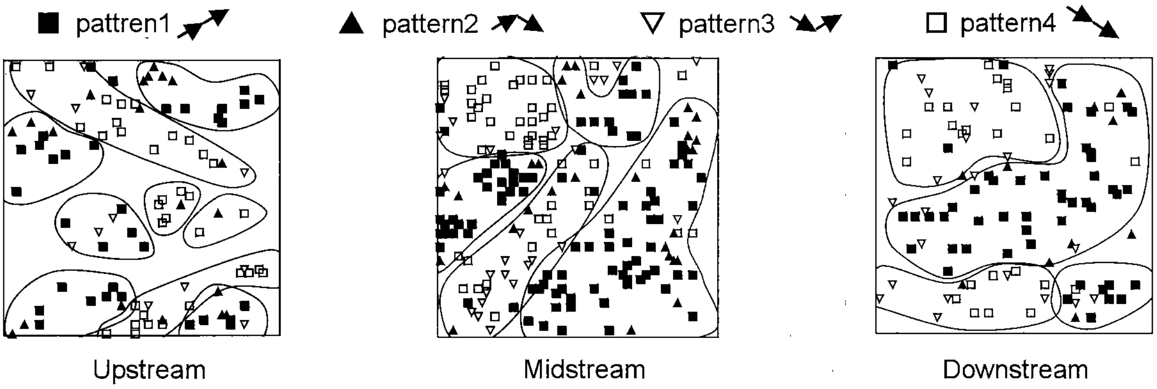

Fig. 10 Distribution of entropy change-patterns with time in the three locations in Fig. 8

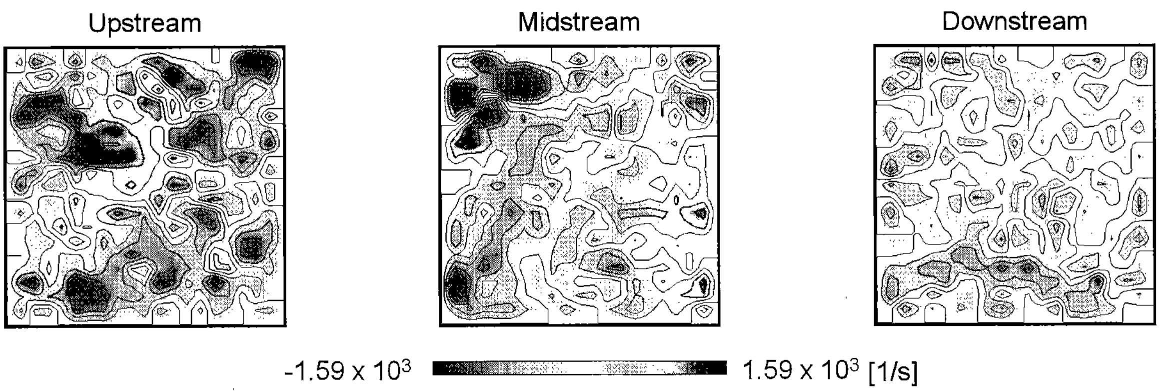

Fig. 11 Vorticity distributions in the three locations in Fig. 8

Table 2 Average entropy and change

		Upstream	Midstream	Downstream
S^*		0.9678	0.9774	0.9870
ΔS_1	+	0.0038	0.0030	0.0009
	−	-0.0023	-0.0037	-0.0006
ΔS_2	+	0.0019	0.0012	0.0009
	−	-0.0036	-0.0017	-0.0006

entropy change pattern indicates that the result is not caused by errors in the analysis. There are many points of decreasing entropy, although ideally the entropy should not decrease. The discrepancy of the decreasing entropy may be caused because the analysis is made on a laser-sheet section although the diffusion is three-dimensional. The cluster of patterns divides into smaller groups in the upstream region and they become larger in the downstream direction. This is similar to the results in Fig. 9.

Table 2 shows the overall entropy of each window in Fig. 8 together with the averaged entropy change. The ΔS_1 is the entropy change from the first to the second picture of the three pictures, and ΔS_2 is the change from the second picture to the third. Averaging was made for each of the positive and negative changes. The entropy values in the three regions show that the diffusion progresses from upstream to downstream, and the entropy increases in that direction. The entropy change is also larger upstream than downstream, indicating a larger upstream diffusion intensity. The absolute values of the positive and negative entropy changes are almost the same.

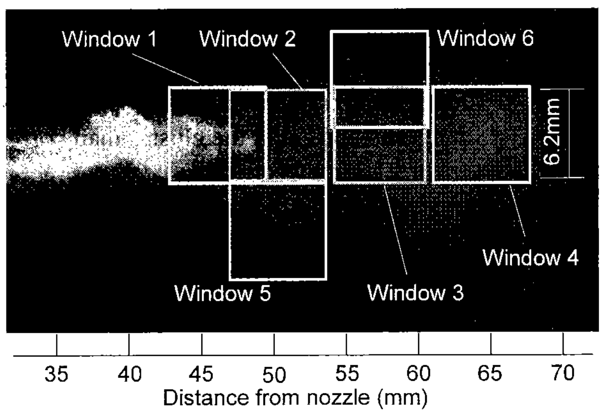

Fig. 12 Location and size of analyzed windows in the diesel spray

6. Analysis of Diesel Sprays in a High Pressure Atmosphere

This section conducts an analysis of diesel sprays injected in a high-pressure atmosphere similar to the tests above. Gas oil was injected into a constant volume bottle from a diameter 0.23mm nozzle. The injection system is a common-rail type, where the injection timing and duration are controlled electronically, the injection pressure was 60MPa. The test section of the constant volume bottle is 100mm in diameter and 30mm thick. It is filled with nitrogen at a pressure of 2MPa. A Nd:YAG laser sheet is introduced from the observation window, it is deflected 90 degrees by a prism, passing through the spray, and going out from the window through a prism. Double pulls of the second harmonic of the Nd:YAG laser was irradiated at time interval of 30μs, and pictures were taken on separate frames with a CCD camera for PIV analysis.

Figure 12 is a picture of the spray at 1.0 ms after start of injection. The areas 1 to 6 were analyzed; areas 1 to 4 are on the axis of the spray, and 5 and 6 locate at the sides of the spray. The distance from the nozzle-exit is indicated at the bottom of the picture and all ar-

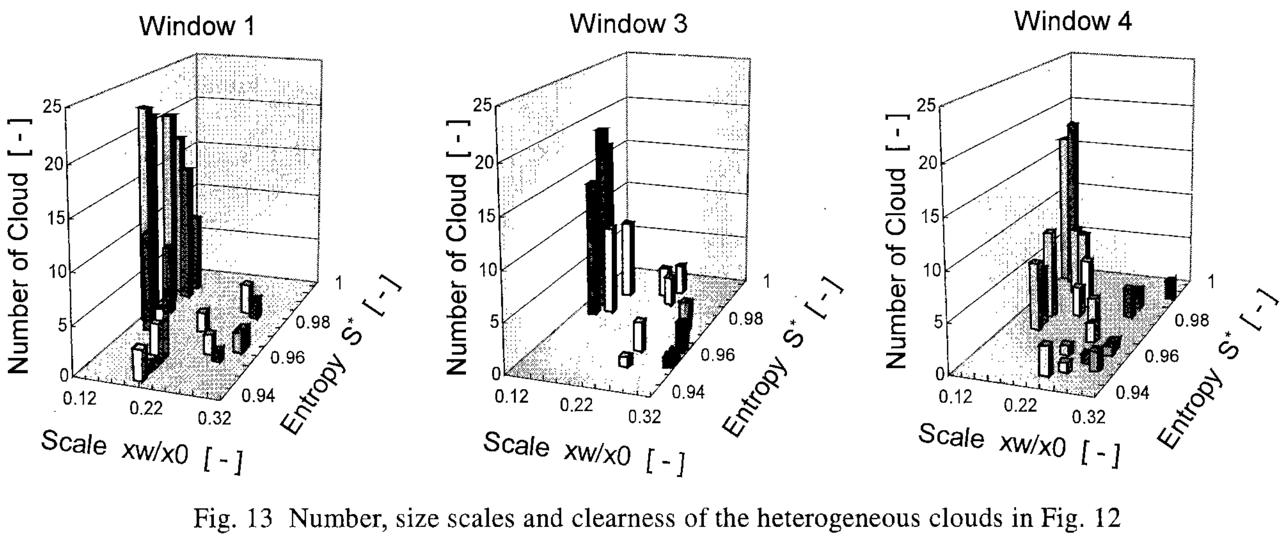

Fig. 13 Number, size scales and clearness of the heterogeneous clouds in Fig. 12

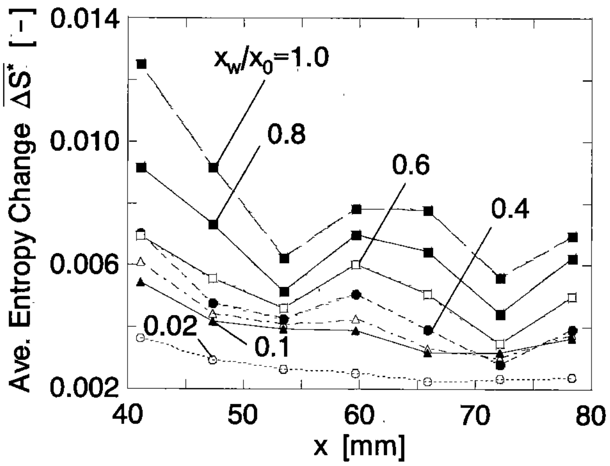

Fig. 14 Averaged entropy changes along the spray axis

eas were 6.2mm square.

Figure 13 is the part of the result of an analysis on the size scale, entropy, and number of clouds, similar to the previous section. The size scale is normalized by the window size, 6.2mm. In this figure, fuel clouds with entropy values above 0.99 were disregarded to show the heterogeneous clouds more clearly. Different size scales appear periodically in all the areas, and there is a hierarchical structure in the heterogeneity. On the spray axis, the number of small-scale heterogeneities decrease and the entropy value at this scale increase from Window 1 to 3. The number of large scale heterogeneities increase and remain at smaller entropy value when moving from area 1 to 4. This indicates that the small, clear upstream fuel clouds gradually shift to larger size scales and that the heterogeneity of the larger size scale diffuses slower than the smaller size scales. The characteristics are similar to the result of the water jet presented in the previous section.

Entropy changes between two frames were analyzed with double-pulsed pictures and Lagrangian tracing in the same manner as in the previous section. Figure 14 is the result of the analysis. The abscissa is the distance from the nozzle, and the ordinate is the averaged entropy change at the section. In the analyzed window, a variety of size scales of sub-windows are selected, and the entropy is calculated in each of the first picture and the corresponding area in the second picture in a Lagrangian manner. The x_w/x_0 is the subarea size of the entropy analysis relative to the area of analysis space, 6.2mm. For any x_w/x_0 size scale in the figure, the entropy change generally decreases as the x distance increases, indicating decreased diffusion intensity downstream in the spray. However there are places where the entropy change is lower than the general declaration curve at 55mm and 70mm from the nozzle. This appears to indicate that there was intermittent slower diffusion regions in the spray direction. The mechanism of the slower diffusion spots will be investigated in further research.

7. Conclusions

Based on the entropy concept of statistic dynamics, a method to analyze the homogeneity degree of a field was developed as a tool to investigate phenomena in diffusion processes. Extending the entropy characteristics, a method to identify the size scales of heterogeneous clouds was also developed. The method was applied to the analysis of heterogeneous structures in liquid jets injected in water and diesel sprays injected into a high-pressure atmosphere.

The results of the analysis showed that the injected liquid breaks into a number of small groups of clouds upstream in the spray, and that the size scale of heterogeneities gradually increase forming a hierarchical structure of heterogeneity as it moves downstream. In the downstream region, small-scale structures diffuse and become unclear, while large scale structures clearly remains. The diffusion intensity is stronger upstream and becomes weaker downstream. However, relatively weak diffusion points appear intermittently along the spray axis. Cluster regions of the same entropy change-pattern distribute as islands, and the cluster map corresponds to the distribution of vorticity.

The analysis method gives new information on diffusion characteristics, and will be useful for research on the mixing of fuel and air in diesel combustion processes.

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