DSC-Vol 44, Transportation Systems — 1992
ASME 1992

A SIMPLE MODEL OF STEERING-CONTROLLED BANKING VEHICLES

Dean Karnopp and Chih Fang
Department of Mechanical, Aeronautical and Materials Engineering
University of California
Davis, California

Abstract

Vehicles which bank in turns include bicycles and motorcycles as well as some trains and proposed commuter vehicles. In this paper, those banking vehicles are considered in which at speed at least, the tilt angle is controlled by steering the front or the front and rear wheels. A model of the dynamics of tilting is derived relating steer angles to the body tilt angle using the assumption of negligible tire slip angles to relate turn geometry to accelerations. A simple control scheme involving tilt angle shows the necessity of counter steering before a turn and is related to the reverse action associated with non-minimum phase systems. The possible benefits to front and rear wheel steering as has been proposed for racing motorcycles are explored. Possible adverse reactions of steering on actively tilted vehicles such as "half lane" commuter vehicles are discussed.

Introduction

There are a number of vehicles which tilt or bank toward the inside of turns in the manner of an airplane when it is executing a coordinated turn. Among ground vehicles, bicycles and motorcycles are obvious examples but in addition several advanced trains tilt and there have been several proposals for tall, narrow commuter vehicles which would bank in turns, Li et al., (1968) and Garrison and Pitstick (1990).

Trains and some commuter vehicles can be tilted directly by an actuator and a control system, or for very light vehicles by direct human action, but single track vehicles are tilted by action of the steering system. In this paper, we study vehicles which at speed are balanced and banked through steering action as a motorcycle. This type of tilting could also apply to a three or four wheeled vehicle with a roll axis near ground level or at least well below the vehicle center of mass.

Motorcycles and similar vehicles are dynamically very complex with many degrees-of-freedom and geometric nonlinearities compounding the nonlinearities associated with tires and suspension elements. Existing models are often so complex that insight into the essential control aspects of the vehicles is nearly impossible. See, for example, Roland (1973) or Bos (1986). Simplified linear models of bicycles have been made for educational purposes, Klein (1989).

In this paper elementary nonlinear and linearized models are derived which still are capable of illuminating the basic control problems associated with turning, banking and balancing. The models have been validated by comparing model responses with measured response plots of motorcycles ridden by riders of various skill levels, Rice (1978). There is good qualitative agreement between the experimental results and results of simple vehicle model simulations with an assumed type of lean control system representing the rider action.

A use of these models is to investigate possible benefits of active rear wheel (or rear axle) steering. There is a potential, for example, of relaxing some of the design constraints on racing motorcycles by modifying the tilt dynamics using active rear wheel steering, Cameron (1992). In addition, the model could be used to study the interaction between steering and the control system of direct tilt control vehicles. This interaction could be beneficial if some active steering were present or harmful if the steering simply disturbed the tilting mechanism, Karnopp and Hibbard, (1992).

Development of the Model

Figure 1 shows several important dimensions and variables associated with the model. For a motorcycle, the sketch is to be imagined as existing in the plane of the ground. For a commuter vehicle, the two wheels shown might represent equivalent wheels representing the front and rear axles and the "ground plane" could be imagined as passing through the roll center of the suspension. The length l is the wheel base and a and b are distances from the center of mass location to the front and rear wheels respectively. The velocity components U and V relate to the center of mass location and are related to a coordinate system moving in the ground plane with the vehicle. The coordinates x and y describe the center of mass location in a fixed coordinate frame. The possibly large angle φ represents the orientation of the vehicle with respect to the y-axis.

The steer angles at the front and rear, β_f and β_r respectively will be assumed to be small since the model is intended only for use at relatively high speeds when the turn radius R is much larger than l.

A major simplifying assumption is that the tire slip angles are negligible so that the wheels proceed in the directions they are pointed. This assumption certainly breaks down at very high lateral acceleration and it precludes studies of wobble and weave types of instabilities but it has the great advantage that no tire characteristics are involved in the model. The motion in the ground plane is determined entirely kinematically including the yaw rate r, the lateral velocity V, and the center of mass location slip angle β.

$$r = U(\beta_f - \beta_r)/l, \tag{1}$$

$$V = U(b\beta_f + a\beta_r)/l, \tag{2}$$

$$\beta = (b\beta_f + a\beta_r)/l, \tag{3}$$

in which small angle approximations have been used.

The instantaneous curve radius R is

$$R = l/(\beta_f - \beta_r), \tag{4}$$

and the path of the center of mass location in the ground plane can be found by solving the following equations:

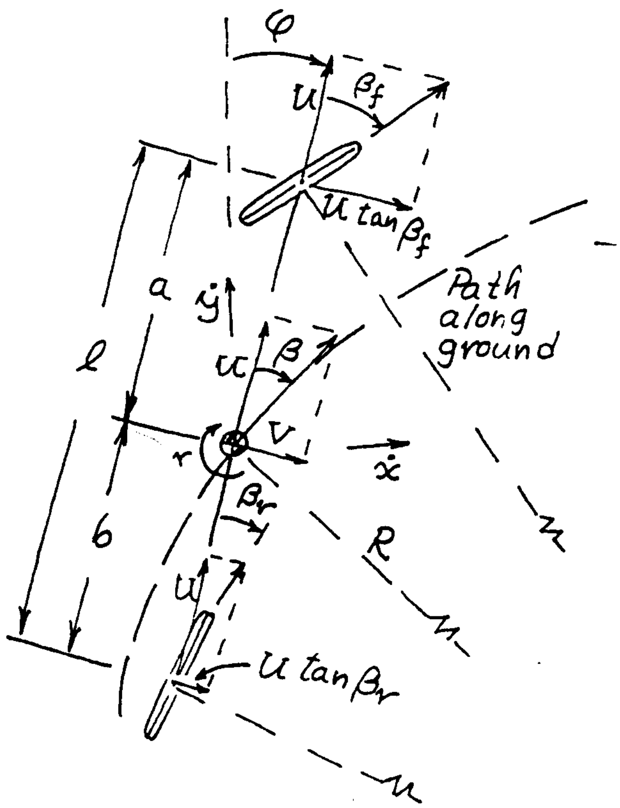

Figure 1. Geometry of motion in ground plane assuming negligible slip angles.

$$\dot{\varphi} = r, \tag{5}$$

$$\dot{y} = U\cos\varphi - V\sin\varphi, \tag{6}$$

$$\dot{x} = U\sin\varphi + V\cos\varphi. \tag{7}$$

In all calculations, we assume that the forward velocity U is constant and large compared to other velocity components.

Figure 2 is a sketch showing the vehicle body with its center of mass a distance h from its ground plane location and tilted at the lean angle θ. The 1, 2, 3 axis system is assumed to be parallel to the body principal axes and the principal moments of inertia relative to the mass center are I_1, I_2, and I_3.

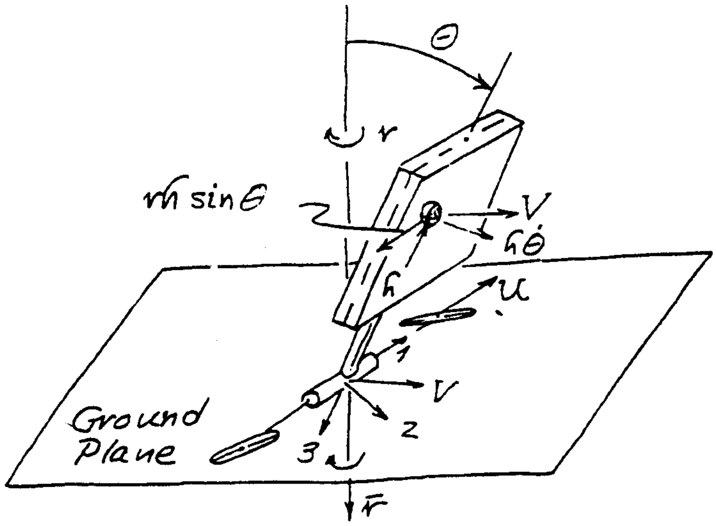

Figure 2. Quantities associated with the degree-of-freedom in lean.

From Fig. 2, one can compute the square of the magnitude of the velocity of the center of mass as

$$v^2 = (U - rh\sin\theta)^2 + (V^2 + h^2\dot{\theta}^2 + 2Vh\dot{\theta}\cos\theta), \tag{8}$$

and the angular velocities as

$$\omega_1 = \dot{\theta}, \quad \omega_2 = r\sin\theta, \quad \omega_3 = r\cos\theta. \tag{9}$$

Using the kinetic energy

$$T = 1/2\,mv^2 + 1/2\,(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2) \tag{10}$$

and the potential energy

$$V_1 = mgh\cos\theta. \tag{11}$$

Lagrange's equation

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}} - \frac{\partial T}{\partial\theta} + \frac{\partial V_1}{\partial\theta} = 0 \tag{12}$$

yields the equation of motion

$$\begin{aligned}(I_1 + mh^2)\ddot{\theta} + (I_3 - I_2 - mh^2)r^2\sin\theta\cos\theta \\ - mgh\sin\theta = -mh\cos\theta(\dot{V} + rU).\end{aligned} \tag{13}$$

The second term in Eq. (13) is almost certainly negligible for all normal maneuvers since it can be combined with the very last term to yield the expression

$$-mh\cos\theta\left[r\left(U+\frac{I_3-I_2-mh^2}{mh}r\sin\theta\right)\right]$$

in which the speed U is clearly dominant in practical cases of interest.

Now, defining a radius of gyration **k** by

$$m k^2 = (I_1 + mh^2) \tag{14}$$

Eq. (13), minus the r^2 term, may be written

$$\frac{k^2}{gh}\ddot{\theta} - \sin\theta = -\frac{U^2}{gl}\cos\theta\left(\frac{b}{U}\dot{\beta}_f + \beta_f + \frac{a}{U}\dot{\beta}_r + \beta_r\right), \tag{15}$$

upon use of Eqs. (1) and (2).

Finally, defining the parameters

$$\frac{k^2}{gh} = \tau_1^2, \quad \frac{b}{U} = \tau_2, \quad \frac{a}{U} = \tau_3, \quad \frac{U^2}{gl} = K, \tag{16}$$

The basic nonlinear equation becomes

$$\tau_1^2\ddot{\theta} - \sin\theta = -K\cos\theta(\tau_2\dot{\beta}_f + \beta_f + \tau_3\dot{\beta}_r - \beta_r). \tag{17}$$

A linearized version of this equation valid for small lean angles is

$$\tau_1^2\ddot{\theta} - \theta = -K(\tau_2\dot{\beta}_f + \beta_f + \tau_3\dot{\beta}_r - \beta_r). \tag{18}$$

For purposes of simulation and control system analysis, it is useful to have a state space version of the equations of motion in which the input variables β_f and β_r appear but not their derivatives. For the linear equation (18), the state equations

$$\dot{\theta} = \frac{1}{\tau_1^2}(x_2 - K(\tau_2\beta_f + \tau_3\beta_r)) \tag{19}$$

$$x_2 = \theta - K(\beta_f - \beta_r) \tag{20}$$

are equivalent to Eq. (18) as can be verified by differentiating Eq. (19) and using Eq. (20). This form is similar to the observability canonical form, Kailath (1980). Figure 3 shows a block diagram of the model in this form.

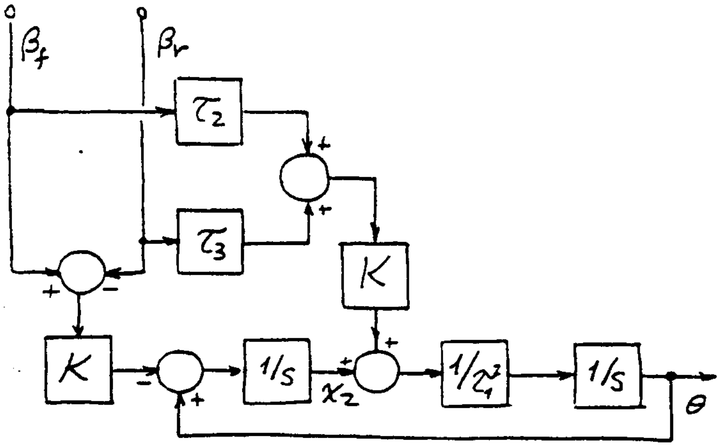

Figure 3. Block diagram for linearized model.

In a similar way, state equations for the nonlinear Eq. (17) can be found.

$$\dot{\theta} = \frac{1}{\tau_1^2}\left(x_2 - K\cos\theta\,(\tau_2\,\beta_f + \tau_3\,\beta_r)\right), \tag{21}$$

$$\dot{x}_2 = \sin\theta - K\cos\theta\,(\beta_f - \beta_r) - (K\sin\theta)\,\dot{\theta}\,(\tau_2\beta_f + \tau_3\,\beta_r), \tag{22}$$

in which Eq. (21) could be used to eliminate $\dot{\theta}$ in Eq. (22). The block diagram for Eqs. (21) and (22) is considerably more complex than the one shown in Fig. 3 but by differentiating Eq. (21) and using Eq. (22) one can see that Eq. (17) results.

Analysis of the Model

The fundamental nature of steering controlled tilting can be simply understood by analyzing the linearized model represented by Eq. (18) or Eqs. (19) and (20) and beginning with front wheel steering only, $\beta_r = 0$. The transfer function between lean angle θ and steer angle β_f is

$$\frac{\theta}{\beta_f} = -\frac{K(\tau_2 s + 1)}{\tau_1^2 s^2 - 1}. \tag{23}$$

Notice that the basic instability of an upside down pendulum is shown by the negative sign in the denominator and the numerator shows that θ is affected not only by β_f but its rate of change.

In order to stabilize the lean angle, we consider a simple proportional control

$$\beta_f = -G(\theta_d - \theta), \tag{24}$$

in which G is a gain, θ_d is a desired lean angle and the minus sign compensates for the minus sign in Eq. (23). A block diagram for the controlled system is shown in Figure 4. Equation (24) might be thought of as a primitive model of the control a motorcycle rider must exert in order to stabilize his machine and to make it negotiate a curve.

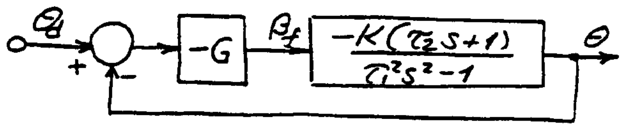

Figure 4. Proportional lean control system.

Note from Eq. 23 that in the steady state, the lean angle and the front steer angle are related by

$$\theta_{ss} = K\beta_{ss} = \frac{U^2}{gl}\beta_{fss} \tag{25}$$

where Eq. (16) has been used. Similarly, the steady turn radius and yaw rate are determined by the steady steer angle using Eqns. (1) and (4), for the front steer only case.

$$r_{ss} = \frac{U\beta_{fss}}{l}, \quad R_{ss} = l/\beta_{fss}. \tag{26}$$

In this sense, the desired lean angle θ_d is related to a desired yaw rate and turn radius at least in the steady state turn. The control system of Fig. 4 and Eq. (24) represents an inner loop required to stabilize the vehicle. The rider (or another control system loop) must provide θ_d as a result of a path following or yaw rate command.

The closed loop transfer function of θ is readily derived from Fig. 4 or Eqns. (23) and (24).

$$\frac{\theta}{\theta_d} = \frac{GK(\tau_2 s + 1)}{\tau_1^2 s^2 + GK\tau_2 s + (GK - 1)} \tag{27}$$

The system is clearly stable only if

$$GK = \frac{GU^2}{gl} > 1 \tag{28}$$

which indicates the difficulty of balancing the vehicle at very low speeds with very large values of gain G.

Another interesting transfer function involves the steer angle

$$\frac{\beta_f}{\theta_d} = \frac{-G(\tau_1^2 s^2 - 1)}{\tau_1^2 s^2 + GK\tau_2 s + (GK - 1)}. \tag{29}$$

For a constant θ_d, the steady lean angle (assuming the system is stable) is:

$$\theta_{ss} = \frac{GK}{GK - 1} = \frac{(GU^2/gl)\,\theta_d}{GU^2/gl - 1} \tag{30}$$

and the steer angle is

$$\beta_{fss} = \frac{G\theta_d}{GK - 1} = \frac{G\theta_d}{GU^2/gl - 1} \tag{31}$$

For a given gain, as speed increases the steady lean angle approaches the commanded lean angle, and the steer angle decreases.

On the other hand, for a step change in θ_d, Eq. (29) shows that the initial steer angle

$$\beta_f(0) = -G\theta_d. \tag{32}$$

Comparing Eqs. (32) and (31) we see that the initial response of θ_f is in the opposite direction to its steady state response, see Fig. 5. In automatic control terms, this reverse action response is a consequence of the non-minimum phase transfer function of Eq. (29) with one zero in the right half of the s-plane. In bicycling and motorcycling circles the initial negative steer angle required for a sharp turn is called "counter steering". One must, of course, remember that the step response is merely used by control engineers to characterize a linear system. For a real vehicle, a step in θ_d could not produce a jump in β_f as implied by Eq. (24). The reverse action type of response does however occur whenever a rapid change in direction is required and some motorcycle riding instructors encourage their students to understand and use the counter intuitive counter steering phenomenon.

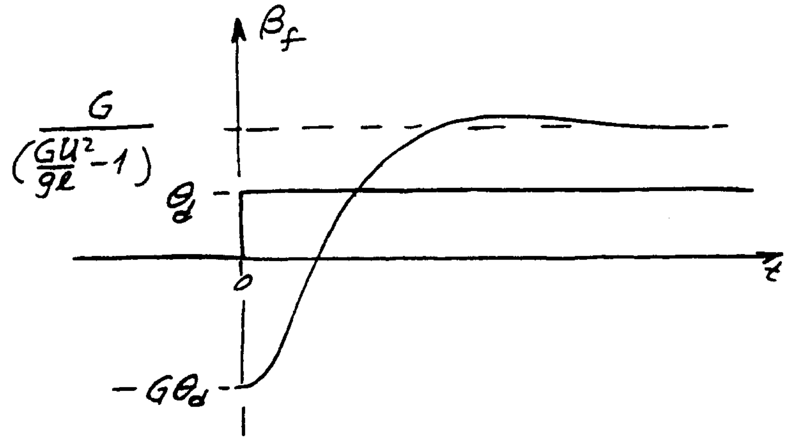

Figure 5. Step response of front wheel steer vehicle steer angle.

Having demonstrated that the simple model illustrates some well known phenomena associated with conventional steering controlled banking vehicles, we can consider useful ways in which rear axle steering might be incorporated. The idea is to modify the tilt dynamics by steering both front and rear wheels in some way. We will illustrate only two open loop strategies although closed loop stability augmentation schemes involving feedback of sensed variables such as yaw rate, lateral acceleration or tilt rate can also be studied.

We first consider simple proportional steer

$$\beta_r = \alpha\,\beta_f, \qquad 0 < \alpha < 1 . \tag{33}$$

Using Eq. (18), the result is

$$\tau_1^2\,\ddot{\theta} - \theta = -K\left[(\tau_2 + \alpha\,\tau_3)\,\dot{\beta}_f + (1-\alpha)\,\beta_f\right] \tag{34}$$

in which we note that the effect of steer angle is reduced and the effect of angular rate is increased. Using the proportional control scheme of Eq. (24) again, we find closed loop transfer functions.

$$\frac{\theta}{\theta_d} = \frac{GK\left[(\tau_2 + \alpha\,\tau_3)s + (1-\alpha)\right]}{\tau_1^2 s^2 + GK(\tau_2 + \alpha\tau_3)s + \left[GK(1-\alpha) - 1\right]}, \tag{35}$$

$$\frac{\beta_f}{\theta_d} = -\frac{G(\tau_1^2 s^2 - 1)}{\tau_1^2 s^2 + GK(\tau_2 + \alpha\tau_3)s + \left[GK\,(1-\alpha) - 1\right]}. \tag{36}$$

When these results are compared with Eqs. (27) and (29) several interesting trends can be seen. For example, the damping term in the denominator polynomial increases with α as does the s-term in the numerator of θ/θ_d. On the other hand, as $\alpha \rightarrow 1$, the system cannot be stabilized except by large gains and steer angles. This correlates with the idea from Eq. (1) that as $\alpha \rightarrow 1$, the yaw rate vanishes and the centrifugal force needed to stabilize the vehicle at a nonzero lean angle also vanishes.

The idea that rear wheel steering might be useful during the initial phases of a sudden turn but not during a nearly steady turn has been used in some automobiles. For tilting vehicles it could be speculated that quickening the counter steering phase but leaving the steady turn phase unchanged might be useful. A way to do this would be to control the rear steer angle dynamically with a so-called washout filter. The transfer function for this open loop controller is

$$\frac{\beta_r}{-\beta_f} = \frac{\alpha \tau_4 s}{\tau_4 s + 1}. \tag{37}$$

In this case, sudden changes of β_f result in the same initial response for β_r Eq. (32). However, if β_f remains constant over a time period long compared to τ_4, β_r will return to zero. Using Eq. (18) the transfer function between θ and β_f is then

$$\frac{\theta}{\beta_f} = \frac{-K\left[(\tau_2 s + 1)(\tau_4 s + 1) + (\tau_3 s - 1)\alpha \tau_4 s\right]}{(\tau_4 s + 1)(\tau_1^2 s^2 - 1)}. \tag{38}$$

Although one can continue on to study the closed loop dynamics analytically, we will simply show some computer simulation results for this control strategy.

Typical Computer Simulation Results

To illustrate the use of the model, some results of simulations of a motorcycle executing a lane change maneuver are shown. The responses correspond roughly to those in Rice (1978). That paper includes both experimental results and a computer simulation plot using an eighth order model. Although the present model is only second order its responses correspond quite well with the results of the eighth order model and agree fairly well with the experimental results, although riders with various skill levels naturally produced different patterns of response.

Figure 6 shows a base case using front steer only. In all cases an assumed desired lean angle, θ_d, shown in Fig. 6 was used, and the speed was 35 mph. Note that θ follows θ_d quite well. The steer angle β_f shows the typical counter steer phase initially and the path x shows a final deviation of 7.19 m

Figure 7 shows the responses when proportional rear steering is incorporated. Equation (33) applies with $\alpha = 0.5$. Note that the proportional controller is able to make θ approximately equal θ_d in this case also, but the steer angle β_f is larger than in Fig. 6. This can be explained by noting from Eq. (1) that the introduction of rear steer has reduced the yaw rate so a larger steer angle is needed to generate a centrifugal force righting moment during a fairly steady part of the turns. On the other hand, the shape of the steer angle plot seems smoother near the transition from counter steer to normal steer. In addition, the final position is 7.91 m so that θ_d and β_f could be reduced slightly to make the same lane change maneuver.

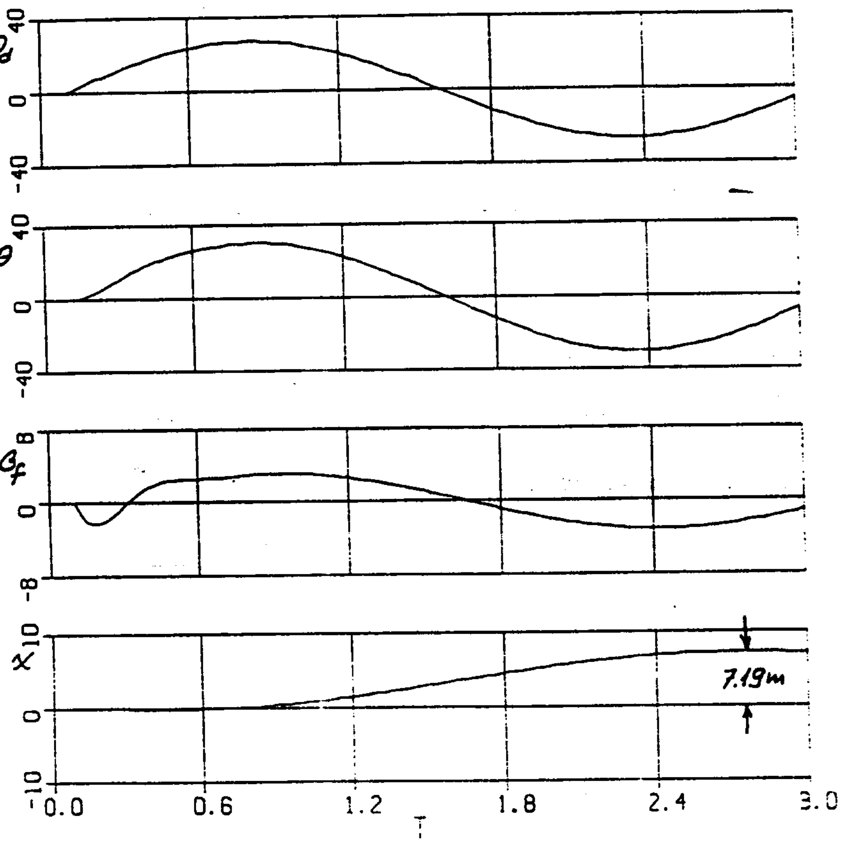

Figure 6. Simulation results for a front wheel steering motorcycle.

Figure 8 incorporates the washout filter effect of Eq. (36) which lets the rear steer angle return to zero for steady turns. Again, the desired lean angle is closely followed and the final path deviation is almost identical to that in the base case. The steer angle β_f is smoother than for the front steer case and is not as large as for the proportional rear steer case. The rear steer angle is initially almost identical to the rear steer angle in the proportional case, but later is considerably smaller.

These results are intended only to show the utility of the elementary model to evaluate possible control strategies. A definitive answer to the question of the utility of various rear wheel steering schemes and tilt control strategies will require further studies and experimental programs.

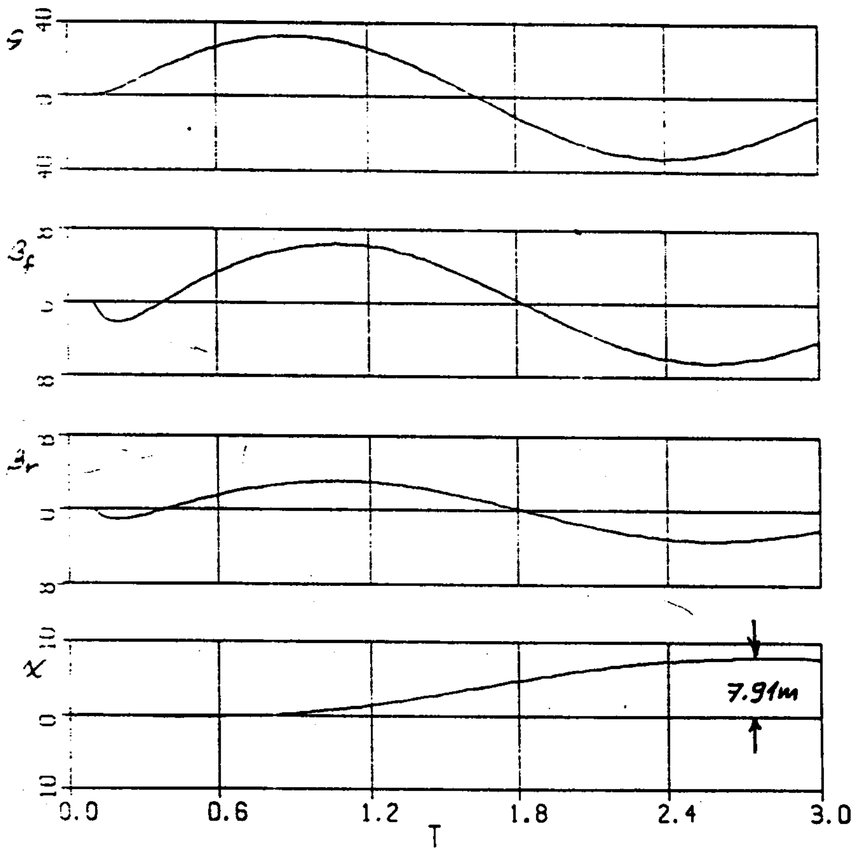

Figure 7. Simulation results for a motorcycle with proportional rear wheel steering.

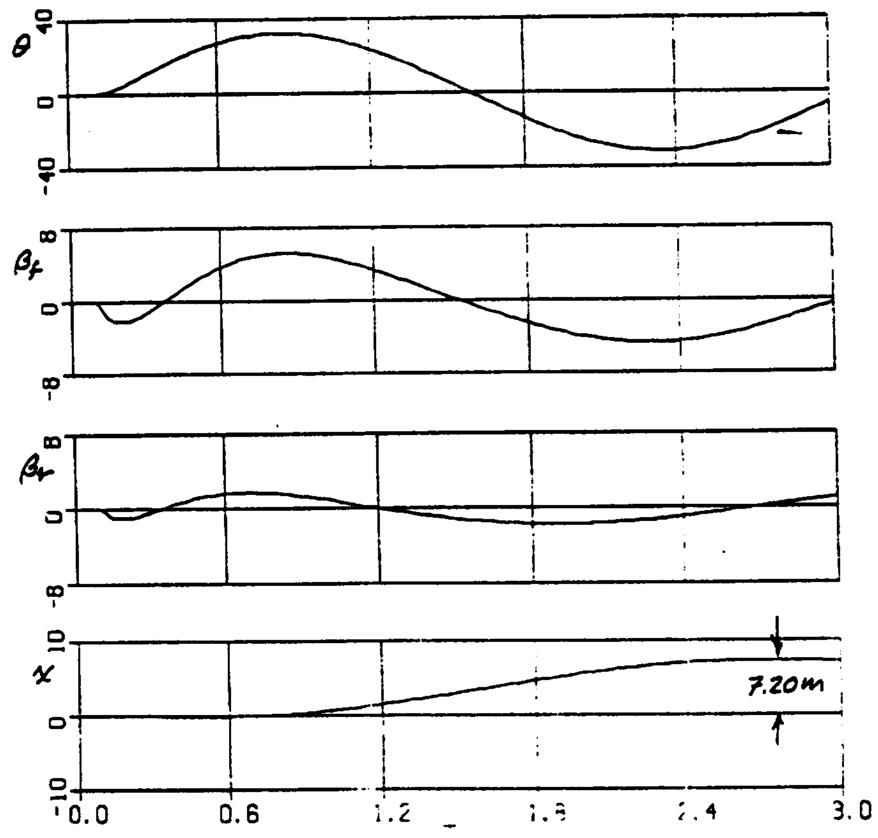

Figure 8. Simulation results for a motorcycle with "washout filter" rear wheel steering.

Conclusions

An extremely elementary model of a steering controlled banking vehicle has been developed. In contrast to more accurate but generally much more complex models, this model can illustrate some fundamental phenomena such as the countersteering effect in a clear way. Surprisingly, the model can produce response plots very similar to plots of experiment results from actual motorcycles.

The model can be used to explore unconventional vehicle concepts such as front and rear wheel steered vehicles and because of the simplicity of the model insight into possible useful control strategies can be developed.

For vehicles with direct tilting mechanisms, the model shows how steering inputs may generate disturbing torques. At the same time, the model could be used to study how steering and direct tilting systems could be coordinated to reduce the load on the tilting mechanism by modifying the steer angle as commanded by the driver. This would, of course, require some sort of steer-by-wire system.

Promising schemes developed using the simple model should be tested and further developed using either the existing complex mathematical models or prototype vehicles.

References

... A. M., (1986). "Modelling Multibody Systems in Terms of Multibond Graphs with ...plication to a Motorcycle," Ph.D. Thesis, University of Twente, Netherlands.

...meron, K., (1992), "Two Wheel Steering," *Cycle World* 31(2):54-55.

...arrison, W. L. and Pitstick, M. E., (1990), "Lean Vehicles: Strategies for Introduction Emphasizing Adjustments to Parking and Road Facilities," SAE Paper 901484.

...ilath, T., (1980), Linear Systems, Prentice Hall.

...rnopp, D. and Hibbard, R., (1992), "Optimum Roll Angle Behavior for Tilting Ground Vehicles," submitted for presentation at 1992 ASME Winter Annual Meeting.

Klein, R. (1989), "Using Bicycles to Teach System Dynamics," *IEEE Control Systems Magazine*, N. 9.

...i, Y. T., Meiry, J. L., and Roeseler, W. G., (1968), "An Active Roll Mode Suspension System for Ground Vehicles," *Trans. ASME, J. Basic Engineering*, June, pp. 167-174.

Rice, R. S., (1978), "Rider Skill Influences on Motorcycle Maneuvering," SAE Paper 780312.

Roland, (1973), "Computer Simulation of Bicycle Dynamics," Mechanics and Sport, Publication of ASME.