ORIGINS OF CYCLIC DISPERSION PATTERNS IN SPARK IGNITION ENGINES

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The focus of this study was to experimentally investigate the development of nonlinear cyclic dispersion patterns in spark ignition engines. The results indicated a transition from stochastic behavior to noisy nonlinear determinism as equivalence ratio was decreased to very lean conditions. The transition to nonlinear deterministic behavior appeared to occur via a period-doubling bifurcation sequence. The structure of the bifurcation sequence and the level of communication between successive cycles were strongly affected by the level of residual fuel in the cylinder. Experimentally observed patterns were compared with patterns predicted by a recently proposed engine model. The comparison supported the hypothesis that the combustion instability develops as a noisy period-doubling bifurcation. The observation of nonlinear determinism under lean conditions may have important implications for engine diagnostics and control since cyclic dispersion is not a purely random process.

Introduction

Fuel lean operation is desired in spark ignition engines to reduce nitrogen oxides and hydrocarbon emissions as well as improve fuel efficiency. Historically, one of the major constraints to practical lean operation has been the large number of misfires and partial burns. A single misfire is capable of destroying modern catalytic converters. Misfires and partial burns are caused when cyclic variations are large enough to push the local in-cylinder equivalence ratio for a cycle very near to or less than the lean limit. Therefore, minimization of cyclic variations is a key requirement for operating near to or extending the effective lean limit.

Conventional approaches to understanding cyclic variations employ the assumption that combustion fluctuations are stochastic or exhibit a combination of stochastic and linear deterministic behavior [1]. Observed deterministic effects have commonly been referred to as "prior-cycle" effects. However, recent studies have indicated that nonlinear phenomena may underlie prior-cycle effects [2-7]. In an effort to better understand the nonlinear phenomena, Daw et al. [2,3] developed a simple low-order model which predicts that lean combustion instability should occur as a dynamical period-doubling bifurcation sequence, a well understood process in nonlinear systems theory. Bifurcations are explained by the model as the result of residual cylinder gas mixing with the fresh intake charge. Small changes in the cylinder gas composition are capable of producing large changes in combustion because of the highly nonlinear effect of composition on flame speed near the lean limit. Although the basic prior-cycle effect is modeled in a deterministic form by Daw et al., stochastic perturbations to the model parameters are included in an attempt to represent many other complex processes such as in-cylinder mixing and fluctuations in as-injected fuel/air ratio.

The goal of this study was to experimentally investigate the onset of lean combustion instability and the role of residual on communication between successive combustion events from the perspective of confirming that previously unrecognized dynamic patterns exist and that these patterns are consistent (or inconsistent) with the Daw et al. model. New data analysis techniques from nonlinear dynamics and chaos theory were employed to better observe possible deterministic patterns in the experimental results. Specifically, cycle-to-cycle combustion variations were characterized using bifurcation diagrams, return maps, and symbol sequence statistics. These techniques allowed the direct comparison of experimental data sets against each other and against model predictions. The increased understanding of cyclic combustion variations gained during this investigation is expected to be useful in developing new approaches for engine diagnostics and control.

Experimental Apparatus and Procedures

Experiments were conducted on a port fuel injected single-cylinder Cooperative Fuel Research (CFR) engine with a compression ratio of 9.0 and 10° of overlap between the intake and exhaust valves. An intake valve

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with a 180° shroud was used to impart a swirling motion on the fuel/air charge during the induction process.

Fuel was metered using a four-hole production fuel injector and a custom built injector driver. Equivalence ratio was varied by adjusting the injected fuel pulse width, and throttle position was maintained constant to minimize effects on intake air dynamics and volumetric efficiency. The level of residual gas fraction was varied by adjusting the intake-exhaust pressure ratio. A lower intake-exhaust pressure ratio corresponds to a higher residual gas fraction. The relative change in residual level was determined based on the change in intake-exhaust pressure ratio [8]. In order to simplify comparisons with the model, engine spark timing was held constant for experimental bifurcation runs. An engine speed control system was the only feedback control used in these experiments. Engine speed was maintained constant even under very lean conditions when engine behavior was erratic.

The investigation consisted of three groups of experiments. The objective of the first group of experiments was to investigate the development of deterministic structure as a function of equivalence ratio. The experiments consisted of obtaining 500 cycles of crank-angle resolved pressure data for several hundred equivalence ratios. The objectives of the second and third groups of experiments were to investigate the effect of residual level on the level of communication between cycles. The experiments consisted of a detailed equivalence ratio sweep similar to the first group of experiments except at a higher residual level and obtaining 10,000 cycles of pressure data at three lean equivalence ratios and six residual levels. Net heat release was calculated for each cycle using conventional techniques based on the numerical integration of the pressure curve [9].

Data Analysis Procedures

The following section includes a brief description of the analysis methods used for processing the data in this investigation. The reader is encouraged to consult the following references for further details [10-12].

The three principal methods used for characterizing the temporal structure in the cycle resolved data were bifurcation diagrams, return maps, and symbol sequence statistics. Bifurcation diagrams are used to observe the effect of individual parameters on nonlinear dynamic data. The diagrams are produced by repeatedly iterating a dynamical system for fixed parameter values beginning with an arbitrary initial condition. Neglecting the initial transients, the results of several hundred iterates are plotted and then the process is repeated for a new parameter value. The bifurcation diagrams depicted in this investigation reflect changes in heat release as equivalence ratio is varied (all other variables are held constant). This type of bifurcation plot (with a single parameter being adjusted) is referred to in the nonlinear dynamics literature as a co-dimension one bifurcation. The experimental bifurcation diagram was developed by allowing the engine to stabilize, recording several hundred cycles of data, and then incrementing equivalence ratio and repeating the process for the new equivalence ratio.

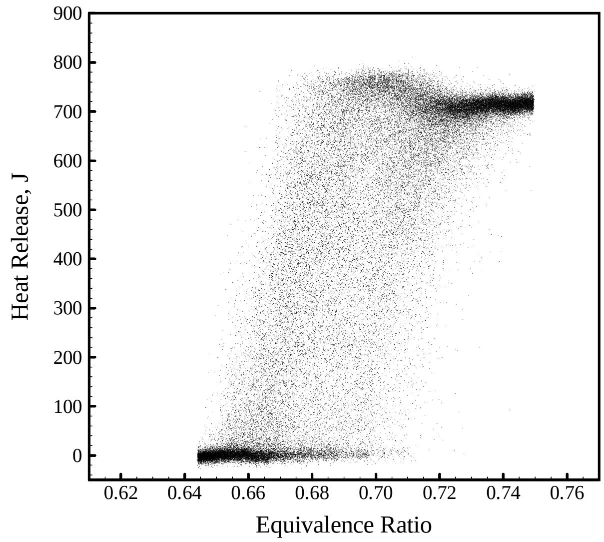
Return maps are commonly used for investigating the structure of nonlinear dynamic data and are typically constructed by plotting an observed variable against itself lagged in time. The return maps presented in this investigation were constructed by plotting the net heat release at cycle $(i+L)$ versus the net heat release at cycle i, where L is the return lag and typically had a value of one. For Gaussian random data, such a map will exhibit a circular, unstructured pattern. A significantly different pattern may indicate the presence of determinism. The appearance of structure in return maps is very robust for low-dimensional dynamics, even in the presence of high levels of noise.

Symbol sequence statistics are also used to investigate dynamical structure in time series data, especially data involving high measurement error or dynamic noise. The basic concept in symbol sequence statistics involves discretizing the original measurements into a limited "alphabet" of possible values. In this investigation a binary partition was applied so our sequences of heat release are converted to sequences of 0's and 1's. The median of the measured values was used as the partition boundary. Measurements less than the median are designated "0" and measurements greater than the median are designated "1". The resulting symbol sequences are then analyzed for non-random combinations of 0's and 1's taken in order. For example, if we look at ordered symbol sequences of length M in the data, any pattern of 0's and 1's that occurs with a frequency significantly greater than $(1/2)^M$ indicates a deterministic temporal relationship. As discussed later, certain types of nonlinear deterministic structure (e.g. bifurcations) produce very distinctive symbol sequence frequency patterns.

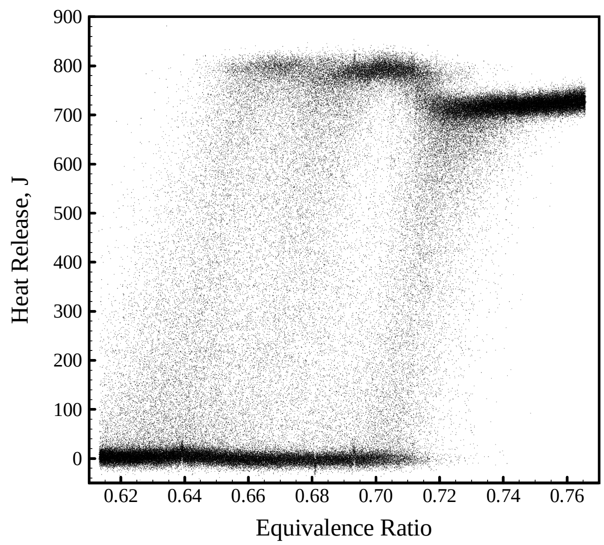
Experimental Results and Discussion

Equivalence ratio was varied in small increments under lean conditions to investigate the effect on cycle dynamics and the results are presented in terms of heat release in the form of bifurcation sequences in Fig. 1. The bifurcation sequences shown in Fig. 1 correspond to a typical residual level in the engine, $r = r_o$, and a significantly higher residual level, $r = 1.8r_o$.

The bifurcation sequence of Fig. 1a appears to reflect five distinct stages of behavior. The first stage ($\phi > 0.75$) corresponds to combustion events which are relatively consistent, with small cyclic fluctuations that appear Gaussian. In the second stage ($0.71 < \phi < 0.75$), the dynamics undergo a transition from relatively consistent combustion events to alternating high-energy and low-energy combustion events that have significantly non-Gaussian structure. The dark band of high-energy combustion events at the top reflects heat release values that



(a)



(b)

Fig. 1. Experimental bifurcation sequences under lean conditions with (a) $r = r_o$ and (b) $r = 1.8r_o$.

are significantly higher than heat release values seen at higher equivalence ratios, suggesting that residual fuel is involved in the combustion events. The third stage ($0.68 < \phi < 0.71$) is dominated by alternating patterns of low-energy and high-energy combustion events. The maximum and minimum heat release values range from the highest value seen for the bifurcation range to zero (total misfire). The fourth stage ($0.65 < \phi < 0.68$) involves a rapid decline in maximum heat release, culminating in a re-convergence of the heat release values. In the final stage ($\phi < 0.65$), combustion ceases or becomes very weak and the heat release fluctuations are again Gaussian-like about a zero mean.

The bifurcation sequence of Fig. 1b corresponds to a higher residual level and appears more complex than the sequence of Fig. 1a. The bifurcation region for the higher residual level is significantly wider and has more distinct stages in dynamic behavior. Specifically, the dynamics undergo a transition at approximately $\phi = 0.68$ to alternating high, medium, and low-energy combustion events which was not seen at a lower residual level. This transition is characteristic of a higher order periodicity in cycle dynamics. Note the dark band of high-energy combustion events seen in Fig. 1a is also present in Fig. 1b. The dark band reflects heat release values significantly higher than heat release values seen at higher equivalence ratios and than heat release values seen in Fig. 1a. The observation of heat release values higher than those seen in Fig. 1a offers more evidence that residual fuel is involved in the combustion events.

Details of the heat release patterns within each bifurcation sequence are obscured somewhat by both dynamic and measurement noise. Dynamic noise is the result of cycle-to-cycle variations in engine parameters such as injected equivalence ratio. Measurement noise is attributed to the noise in the in-cylinder pressure measurement system and errors incurred in calculating heat release. For example, the combustion event under very lean conditions is considerably different than under stoichiometric conditions. Combustion may still be occurring when the exhaust valve opens due to low flame speeds resulting from lean conditions. Combustion after the exhaust valve has opened was not included in the heat release calculation.

Cyclic variations and the effect of residual gas fraction were investigated in more detail by obtaining 10,000 cycles of in-cylinder pressure data for ten equivalence ratios evenly spaced across the equivalence ratio ranges shown in Fig. 1. The large amount of data is necessary to clearly resolve consistent dynamical patterns as opposed to short term transients. Other studies may have missed the patterns presented in this investigation because typical data sets collected in the past have been much shorter. A typical return map for the conditions represented in Fig. 1a is shown in Fig. 2 for $\phi = 0.72$. The general appearance of the return map in Fig. 2 implies some type of strong deterministic connection between successive combustion events. If the dynamics were stochastic-dominated, the return map would have a circular, unstructured pattern. If the dynamics were dominated by a non-Gaussian random process (e.g., a Weibull distributed random process), the resulting pattern would be similar to Fig. 2 but without the negative slope in the upper "arm" and without the strong asymmetry about the 45 degree diagonal. The asymmetry has been seen in the results of other studies [13]. The presence of the asymmetry about the diagonal is very important because it actually reflects an asymmetry in time (i.e., reversing the axes for *i* and *i+1* is the same as looking at the plot for backward time). Such time asymmetry is not possible for Gaussian linear processes

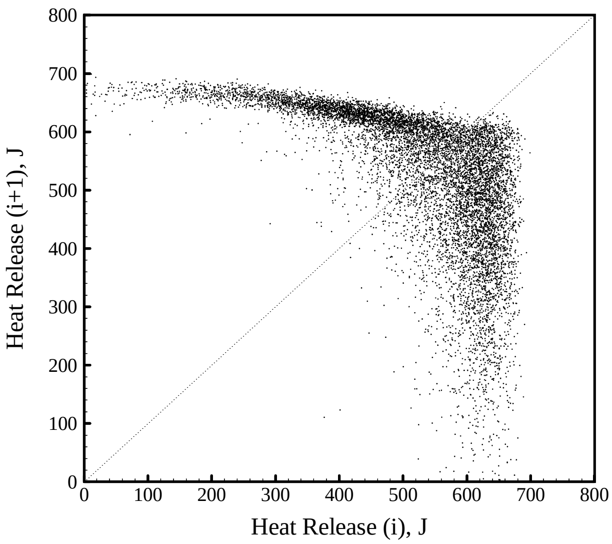


Fig. 2. Experimental return map for $r = r_o$ and $\phi = 0.72$.

and supports the hypothesis that we are observing a truly dynamic nonlinear process (that is a nonlinear process with memory) [14]. In a physical sense, time asymmetry implies that prior-cycle effects tend to follow a specific sequential mechanism; i.e., the likelihood of a misfire followed by a "rebound" combustion event is much higher than the other way around.

The effect of residual gas fraction on communication between successive combustion events was investigated in more detail by obtaining 10,000 cycles of in-cylinder pressure data for three lean equivalence ratios at six residual levels. Symbol sequence statistics were used to observe multi-cycle deterministic patterns in the time series measurements. Specifically, the original measurements were discretized as symbols in the manner described earlier, and then non-random sequences of events were identified by their unexpectedly high frequency of occurrence. Any random process should produce a flat histogram with a symbol partition chosen to give equiprobable occurrence of each symbol.

Symbol sequence histograms for the CFR engine at moderately lean conditions and two levels of residual are shown in Fig. 3. Relative frequency is indicated on the vertical axis and "pattern number" (described below) is indicated on the horizontal axis. The sequence length in this case is six and corresponds to patterns formed by six sequential cycles. The pattern number in the sequence histograms refers to the decimal equivalent of a corresponding binary sequence. For example, the binary patterns 010101 and 101010 correspond to pattern numbers 21 and 42, respectively. These sequences in particular correspond to alternations of low-energy and high-energy combustion events and appear to be most likely of all sequences to occur in the data. These peaks are also consistent with noisy period-two oscillations. Other peaks

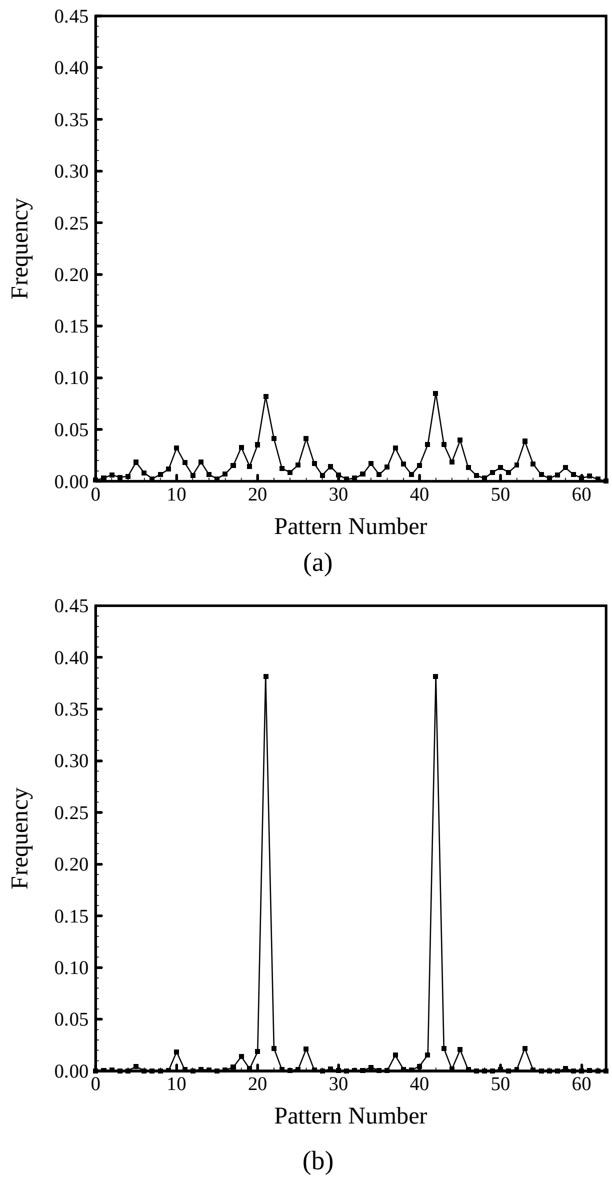


Fig. 3. Experimental binary symbol sequence histograms at $\phi = 0.70$ for (a) $r = r_o$ and (b) $r = 1.8r_o$.

correspond to transitions into and out of the repeated alternating sequences. For example, the dynamics must pass through the sequence 001010 or 101010 to enter the sequence 010101. The relative peak heights are useful for describing how frequently or infrequently these transitions occur.

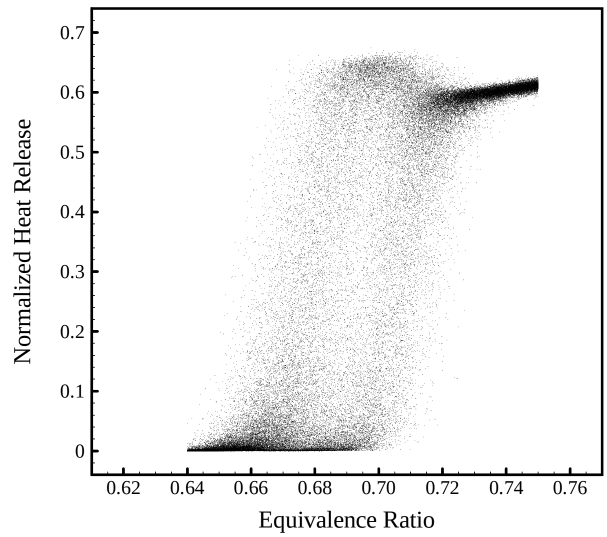
The sequence histograms shown in Fig. 3 indicate the engine undergoes a noisy period-two bifurcation and the level of communication between successive combustion events is strongly related to the level of residual. Specifically, the frequency of alternating low-energy and high-energy combustion events (pattern numbers 21 and 42) increases with the level of residual. The histograms offer more evidence that residual is involved in the combustion events and is likely the dominant means of communication between successive cycles.

Model Results and Discussion

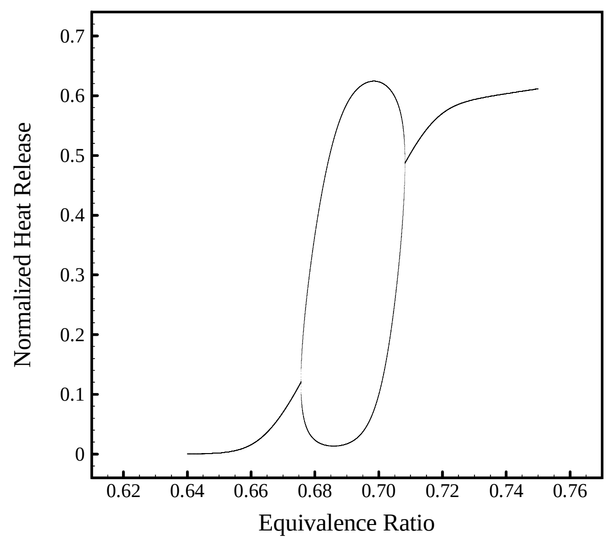
The Daw et al. model was developed to investigate nonlinear cycle dynamics in spark ignition engines under lean conditions and is structured around the interaction between deterministic and stochastic processes. The model is discrete in time, representing each cycle as a single event. The model is very simple and predicts normalized cycle-resolved heat release. The simplicity of the model is a computational advantage in simulating the large number of cycles necessary for characterizing deterministic patterns. Complete details of the model are presented in the references [2,3].

The experimental results were compared to predictions of the Daw et al. model by simulating bifurcation sequences, return maps, and symbol sequence histograms using "fitted" values for the model parameters. Key parameters are cylinder gas residual fraction, r, combustion efficiency curve parameters, ϕ_u and ϕ_l, and dynamic noise levels, σ_ϕ. The parameter values were found by fitting the model to the experimental data of Fig. 1a, constraining the values to stay within reasonable physical limits (e.g., residual fraction should be between 0 and 0.3). The reader is encouraged to consult [4] for more details on the fitting process. Figure 4 illustrates the bifurcation sequences predicted by the model with and without dynamic noise. Figure 4a shows the bifurcation sequence with random fluctuations in inducted equivalence ratio. Note the bifurcation sequence of Fig. 4a is very similar to the experimental sequence of Fig. 1a. The five stages discussed in the experimental sequence are present in the model sequence, although subtle differences between the model and experimental results are present. Specifically, the experimental bifurcation sequence appears noisier than the model results. The differences in noise are believed to be the result of measurement noise in the experimental results. Other subtle differences are due to the simplicity of the model. The model is not able to directly take into account several significant engine parameters such as spark timing, injection timing, and coolant temperature. The bifurcation sequence without dynamic noise for the same model parameters as Fig. 4a is shown in Fig. 4b. A period-two bifurcation occurs without noise, although the Daw et al. model predicts that higher order periodicity and chaos can occur for different engine parameter values. Note the bifurcation region is significantly narrower in the absence of noise. The dynamic noise appears to smear the bifurcation region causing the period-two bifurcation to begin at a higher equivalence ratio.

The similarity of the Daw et al. model to the observed combustion dynamics is also revealed in return maps of the model output. A typical return map for the model



(a)



(b)

Fig. 4. Model bifurcation sequences with (a) $\sigma_\phi = 0.007$, and (b) $\sigma_\phi = 0$. Fixed model parameters are $r = 0.14$, $\phi_u = 0.685$, and $\phi_l = 0.665$.

parameters and noise level used in Fig. 4a is shown in Fig. 5 for $\phi = 0.72$. The model return map pattern agreed well with the experimental pattern shown in Fig. 2 indicating the research engine is following a similar pattern under lean conditions. The strong time asymmetry above and below the diagonal seen in the experimental results in Fig. 2 was also visible in the model results and appeared to be a function of the dynamic noise level. The time asymmetry becomes more prominent at higher dynamic noise levels. Symbol sequence histograms were also compared for the model and experimental results. Although not shown here, the comparison indicated the

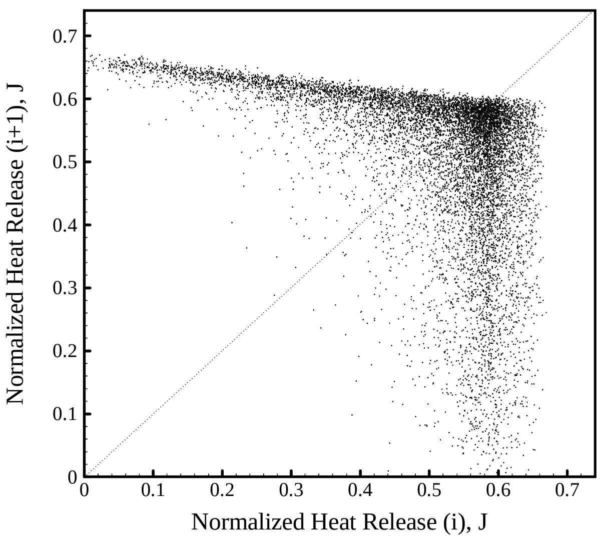


Fig. 5. Model return map for $\phi = 0.72$ and $\sigma_\phi = 0.007$.

strong oscillations in the model are produced by a noisy period-two bifurcation which is smeared out by dynamic noise. The similarity offers more evidence the research engine is following a similar pattern.

The Daw et al. model was also used to investigate the effect of residual level on the stages of the bifurcation sequence. Although not shown here, the model predicted higher order periodicity and chaos at higher residual levels. The model dynamics were very similar to the dynamics seen in the experimental bifurcation sequence of Fig. 1b. Specifically, the transition to the alternating low, medium, and high energy combustion events was present in the model results as well as the widening of the bifurcation region. The effect of residual on the level of communication between successive combustion events was also investigated by the Daw et al. model. The model clearly showed an increase in communication with the level of residual as was seen in Fig. 3. Specifically, the frequency of alternating low-energy and high-energy combustion events increased with residual level in the model.

Summary and Conclusions

The presence of nonlinear deterministic structure in cycle-to-cycle variations from a spark ignition engine under lean conditions was confirmed using data analysis techniques from nonlinear dynamics and chaos theory. The cyclic combustion variations in the single-cylinder research engine experienced a clear transition from stochastic to noisy deterministic behavior as equivalence ratio was reduced to lean conditions. Engine dynamics appeared to pass through distinct stages including stochastic, periodic, and possibly chaotic behavior. The level of residual in the engine appeared to be the dominant means of communication between successive combustion events. A higher level of residual caused an increase in the width of the bifurcation region, higher order periodicity, and the bifurcation sequence to occur at a higher equivalence ratio.

The Daw et al. model predicted the transition from stochastic to noisy deterministic behavior as equivalence ratio was reduced to lean conditions. Specifically, the model predicted the stages of the engine dynamics seen in the experimental results. The effect of residual on the cycle dynamics and level of communication between successive combustion events was also predicted by the Daw et al. model. Although subtle differences were visible between the experimental and model results, the model was able to accurately simulate the dynamical trends in engine behavior. The consistency between the experimental observations and the Daw et al. model provides a good example of the value of low-order models for simulating combustion dynamics. In particular, such models are likely to be extremely useful for real-time diagnostics and controls where the luxury of high computational overhead is simply not available or cost effective. The investigation also indicates that analytical tools from nonlinear dynamics and chaos theory are capable of revealing a large amount of information from engine combustion measurements that have been previously unrecognized by conventional stochastic analysis methods.

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