Robust Predictive Control Design for Automatic Steering

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Abstract. This paper presents a feasibility study of the robust predictive control for the design of an automatic steering system. The involved vehicle is a BMW520i and the feedback is only on the lateral displacement unlike in the available studies where an additional feedback on the yaw rate is commonly used to improve the performance. The robustness feature is particularly motivated by those ubiquitous variations in velocity, tire-road contact and vehicle mass. The control design is performed using a sensitivity function shaping procedure that heavily borrows from the robust control culture. Intensive simulations, using a full-scale nonlinear model of the considered vehicle, are carried out to emphasize the performance of the proposed robust predictive automatic steering system.

Key Words. Automatic steering, Control model identification, Predictive control design, Usual sensitivity function, Robustness

1. INTRODUCTION

Advanced control of automotive vehicle systems is more and more discussed in the open literature and widely developed in industry. International Workshops have been dedicated to the automotive control. It has been pointed out that the automatic steering is of fundamental interest for urban transport vehicles and those automated highway traffics of the next century (Ito *et al.*, 1990; Halanay *et al.*, 1994; Ackermann *et al.*, 1995; Ramirez-Mendoza *et al.*, 1995; M'Saad *et al.*, 1996).

The main purpose of automatic steering consists in performing a robust tracking of a reference path in spite of the uncertain operating conditions in view of large variations in velocity, contact characteristics between tyres and road surface and vehicle mass. The underlying control systems involve feedbacks of both the lateral displacement and the yaw rate and use the front steering angle as a manipulated variable. The reference may consist of permanent magnets in the road or the magnetic field of an electrically supplied wire. The involved tracking error is measured by a displacement sensor mounted in the center of the front end of the vehicle while the yaw rate measurement is performed by a gyro.

The involved control problem represents a challenging opportunity to investigate the advanced control techniques that have reached a reasonable level of maturity. This is mainly a result of many years of effort devoted to the understanding of the control theory as well as the revolutionary progress in the computer technology that makes the implementation of control systems simpler and cheaper. Of particular interest, several feasibility studies have been already reported and are finding their way to promising industrial applications (See for instance (Ackermann *et al.*, 1995) and reference list therein).

In this paper, the authors aim at investigating the applicability of the robust predictive control for automatic steering using solely lateral displacement measurement unlike in the earlier design studies (Ackermann *et al.*, 1995). A realistic simulation framework involving a full-scale nonlinear model of a vehicle BMW520i is used to this end. This simulator has been developed at the Institute of Industrial Information Systems of Karlsruhe, to investigate the automotive control (Majjad *et al.*, 1996). The control design is carried out using an appropriate predictive control approach that has been developed from the generalized predictive control (Clarke *et al.*, 1987) in the spirit of the robust control theory (Limbeer and Green, 1995; Zhou *et al.*, 1995). Besides the standard design specifications, namely maintaining the lateral displacement as small as possible for typical maneuvers taking into account the actuator

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constraints and the passenger comfort, suitable shapings of the usual sensitivity functions have to be achieved. This allows to deal with those ubiquitous variations in velocity, road adhesion factor and vehicle mass. Indeed, the usual sensitivity functions are used as suitable quantifiers for both nominal performance and stability robustness (Limbeer and Green, 1995; Zhou *et al.*, 1995). It is worth mentionning that the adopted methodology requires a good know-how in system identification: the control system and its usual sensitivity functions are determined from control models that should be identified over the domain of possible operating conditions (Ljung, 1987).

The involved experimental evaluation is carried out using the advanced control software package SIMART (M'Saad, 1994) which allows to perform all the steps involved in any genuine control system design, namely the performance specification, the plant control model identification and its validation, the control design, the stability and performance robustness analysis, and the appropriate control design. A suitable parameter adaptation capability as well as a partial state tracking tracking capability can be incorporated into the control algorithm when needed.

2. PREDICTIVE CONTROL APPROACH

In the following, we will precise the main components of the predictive control approach we are concerned with.

The input-output behavior of the plant is assumed to be appropriately approximated by the following backward shift operator (q^{-1}) model:

$$\begin{aligned} A(q^{-1})y(t) &= B(q^{-1})u(t-d-1)+v(t) \\ D(q^{-1})v(t) &= A(q^{-1})C(q^{-1})\gamma(t) \end{aligned}$$

with

$$\begin{aligned} A(q^{-1}) &= 1+a_1q^{-1}+...+a_{na}q^{-na} \\ B(q^{-1}) &= b_o+b_1q^{-1}+...+b_{nb}q^{-nb} \\ C(q^{-1}) &= 1+c_1q^{-1}+...+c_{nc}q^{-nc} \\ D(q^{-1}) &= 1+d_1q^{-1}+...+d_{nd}q^{-nd} \end{aligned}$$

where $u(t)$ is the control variable, $y(t)$ is the measured plant output, d denotes the minimum plant model delay in sampling periods, $v(t)$ represents the external disturbances and $\{\gamma(t)\}$ is assumed to be a sequence of widely spread pulses of unknown magnitude or independent random variables with zero mean values and finite variances.

The predictive control objective consists in minimizing, in receding horizon sense with respect to the vector input $U_f(t+ch-1) = [u_f(t)\dots u_f(t+ch-1)]^T$, the following linear quadratic cost function

$$E\left\{\sum_{j=sh}^{ph}(y_f(t+j))^2+\rho(u_f(t+j-sh))^2\right\}$$

subject to

$$u_f(t+i)=0 \quad \text{for} \quad ch \le i < ph$$

with

$$\begin{aligned} W_{yd}(q^{-1})y_f(t) &= W_{yn}(q^{-1})y(t) \\ W_{ud}(q^{-1})u_f(t) &= D(q^{-1})W_{un}(q^{-1})u(t) \end{aligned}$$

where $E\{\cdot\}$ denotes the mathematical expectation, sh, ph and ch are the starting, prediction and control horizons according to the long range predictive control culture, ρ is a positive scalar and $\mathcal{W}_u(z^{-1}) = \frac{D(z^{-1})W_{un}(z^{-1})}{W_{ud}(z^{-1})}$ and $\mathcal{W}_y(z^{-1}) = \frac{W_{yn}(z^{-1})}{W_{yd}(z^{-1})}$ denote user specified input and output frequency weightings, respectively. The frequency weightings are mainly motivated by stability and performance robustness considerations and are such that all polynomials $W_{xx}(q^{-1})$ are Hurwitz.

The underlying control problem will be handled using the generalized predictive control approach proposed in (Clarke *et al.*, 1987) from the following plant reparametrization:

$$\bar{A}(q^{-1})y_f(t)=\bar{B}(q^{-1})u_f(t-d-1)+\bar{C}(q^{-1})\gamma(t)$$

with

$$\begin{aligned} \bar{A}(q^{-1}) &= A(q^{-1})D(q^{-1})W_{yd}(q^{-1})W_{un}(q^{-1}) \\ \bar{B}(q^{-1}) &= B(q^{-1})W_{yn}(q^{-1})W_{ud}(q^{-1}) \\ \bar{C}(q^{-1}) &= A(q^{-1})C(q^{-1})W_{yn}(q^{-1})W_{un}(q^{-1}) \end{aligned}$$

The resulting controller may be given the following linear form:

$$S(q^{-1})D(q^{-1})u(t)+R(q^{-1})y(t)=0$$

with

$$\begin{aligned} S(q^{-1}) &= \bar{S}(q^{-1})W_{yd}(q^{-1})W_{un}(q^{-1}) \\ R(q^{-1}) &= \bar{R}(q^{-1})W_{yn}(q^{-1})W_{ud}(q^{-1}) \end{aligned}$$

where the polynomials $\bar{S}(q^{-1})$ and $\bar{R}(q^{-1})$ depend on the plant model as well as the design parameters.

The involved nominal control system may be represented as shown in figure 1 where $v_u(t)$ and $v_y(t)$ denote respectively the input and output disturbances, and $\nu_u(t)$ and $\nu_y(t)$ are the input and output noise measurements. The corresponding input-output behavior is described by

$$\begin{aligned}
P_c(q^{-1})y(t) &= q^{-d-1}B(q^{-1})S(q^{-1})D(q^{-1})v_u(t) \\
&+ q^{-d-1}B(q^{-1})S(q^{-1})D(q^{-1})\nu_u(t) \\
&+ A(q^{-1})S(q^{-1})D(q^{-1})v_y(t) \\
&- q^{-d-1}B(q^{-1})R(q^{-1})\nu_y(t)
\end{aligned}$$

$$\begin{aligned}
P_c(q^{-1})u(t) &= A(q^{-1})S(q^{-1})D(q^{-1})v_u(t) \\
&- q^{-d-1}B(q^{-1})R(q^{-1})\nu_u(t) \\
&- A(q^{-1})R(q^{-1})[v_y(t) + \nu_y(t)]
\end{aligned}$$

where $P_c(q^{-1})$ is the characteristic polynomial which can be factored as follows

$$P_c(q^{-1}) = P_f(q^{-1})P_o(,q^{-1})$$

where $P_f(q^{-1})$ and $P_0(q^{-1})$ denote the characteristic polynomial of the underlying receding horizon linear quadratic control and state predictor, respectively. Notice that the predictor dynamics could be chosen bearing in mind the optimal estimation theory, namely $P_o(q^{-1}) = C(q^{-1})$.

Of particular importance, the control system is asymptotically stable if and only if the characteristic polynomial is Hurwitz.

$$P_c(q^{-1}) = 0 \quad \Rightarrow \quad |q| < 1$$

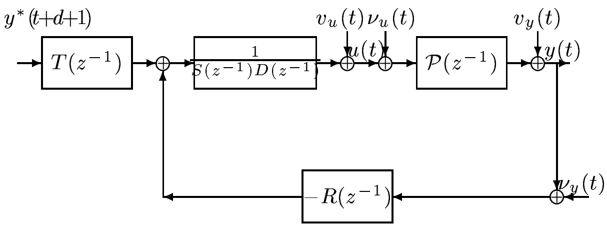

Fig. 1. Nominal control system

The control system performance may be represented as shown in figure 2 and figure 3, where $\mathcal{P}(z^{-1})$ and $\mathcal{R}(z^{-1})$ denote the control design model and its underlying regulator respectively, and $\mathcal{S}(z^{-1})$ and $\mathcal{T}(z^{-1})$ are respectively the sensitivity and complementary sensitivity functions, i.e.

$$\mathcal{S}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})D(z^{-1})}{P_c(q^{-1})}$$

and

$$\mathcal{T}(z^{-1}) = \frac{z^{-d-1}B(z^{-1})R(z^{-1})}{P_c(q^{-1})}$$

The nominal performances of the control system as well as the stability robustness can be evaluated from its usual sensitivity functions relating the exogenous input to the plant input and output, respectively (M'Saad *et al.*, 1996). The shapes of the sensitivity functions may be refined by properly specifying the involved design parameters. To do so, an iterative procedure is needed and hence a useful CACSD software package.

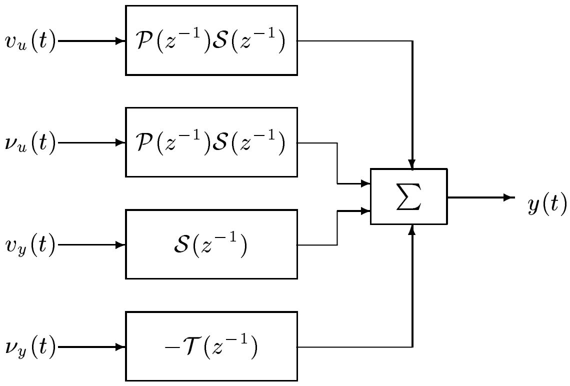

Fig. 2. Nominal output performance

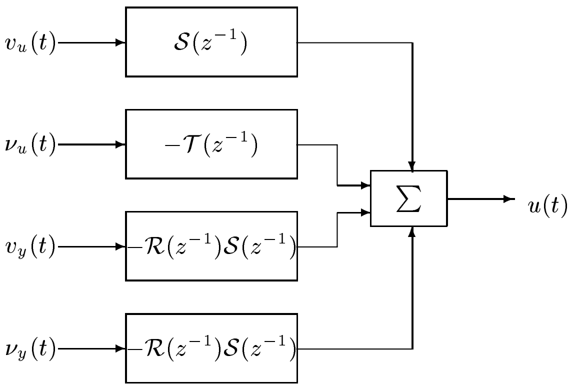

Fig. 3. Nominal input performance

3. EXPERIMENTAL EVALUATION

The design of automatic steering systems is usually based on a linear model of the vehicle dynamics. The classical single-track model is commonly used for this purpose. In the present feasibility study, a nonlinear vehicle model, which has been developed at the Institute for Industrial Information Systems of the Karlsruhe Technical University, was used. This consists of several sub-models, e.g. tyres, chassis, suspension, steering system, system aerodynamics and road, and takes all important nonlinearities into account. The data has been adapted to a BMW520i and proved to demonstrate a realistic behavior during various driving conditions, namely normal to critical situations at the limit of stability (Majjad *et al.*, 1996). Figure 4 shows the vehicle and

Fig. 4. Vehicle and pertinent variables

the variables of interest. The design specifications have been adapted from those considered in earlier design studies (Ackermann *et al.*, 1995). They are primarily given in terms of actuator constraints and passenger comfort as follows:

- The steering angle and its rate should satisfy $|\delta(t)| \leq 40$ deg and $|\dot{\delta}(t)| \leq 23$ deg/s.
- The displacement from the guideline must not exceed 15 cm in transient behavior and 2 cm in steady state behavior.
- The yaw rate should not exceed 4 deg/s for reasonnable speed and 8 deg/s for high speed.

The reference maneuver considered for the simulation consists in a transition from a straight line into a circular one with a radius of 400 m.

As the predictive control approach, we are concerned with, is based on linear control design model, the vehicle dynamics has been identified over an important domain of driving conditions from linear model strucures using the system identification approach of the advanced control software package SIMART. More precisely, the identification has been performed using a least squares method incorporating suitable signal filtering, data normalization, adaptation gain regularization and any prior knowledge. The data were the steering angle and the second derivative of the the displacement between sensor and reference path. The latter has been determined using measurable signals, namely the lateral acceleration of the vehicle and the second derivative of the yaw angle, as follows

$$\ddot{y}_S(t) = (a_y(t) + l_s\ddot{\psi}(t))\cos\psi \simeq a_y(t) + l_s\ddot{\psi}(t)$$

Such a formula is valid only for a straight path which has been considered for all identification experiments.

The control design models have been obtained from the identified model by simply multiplying by a double integrator. This provides sets of pulse transfer functions relating the sensor displacement to the steering angle.

Intensive identification experiments have been made to determine a robust control design model using a suitable offset-free binary random sequence. They particularly showed that the crucial variations are those of the velocity and road adhesion factor. The dynamics of the vehicle are not very sensitive to variations in mass. An average mass corresponding to the mass of the vehicle loaded with the driver and one passenger has been considered throughout the experimental evaluation. The Bode plots of the identified

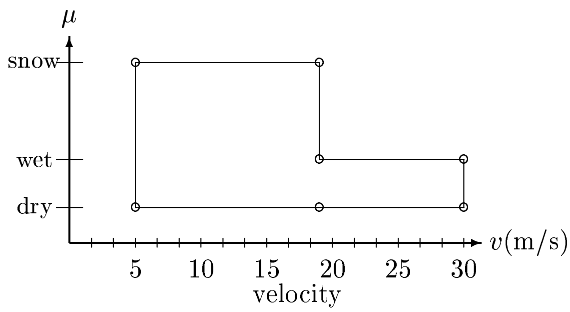

Fig. 5. Operating domain

models obtained for various velocities on a dry road are shown in figure 6. Figure 7 shows the Bode plots of the models on a dry, a wet and on snow road, respectively, for a speed of 15 m/s. Notice that the frequency responses of the identified models are relatively sensitive to the velocity and tyre-road contact. The identified models have been validated in both the time and frequency domains. In the time domain, the identification performance quantifiers are based on correlation function, information likelihood measure and singular values of the identified model. The

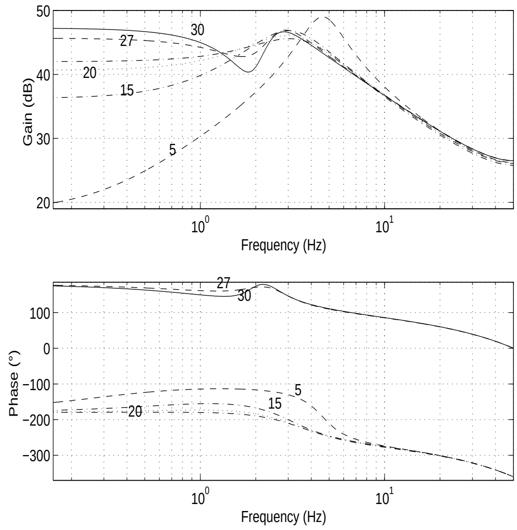

Fig. 6. Frequency responses of identified models (dry road and a varying speed)

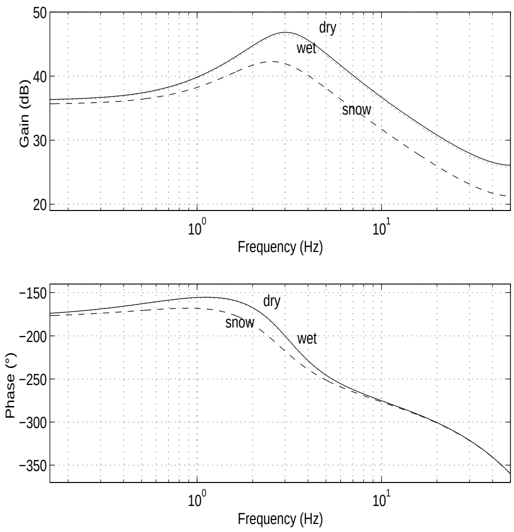

Fig. 7. Frequency responses of identified models (constant speed and varying adhesion factor)

difference between an experimental frequency response and the frequency response of the identified model is used to validate the identification in the frequency domain. The involved experimental frequency response is determined using an appropriate FFT procedure, i.e. $\mathcal{P}_e(j\omega) = \frac{\mathrm{FFT}\{y(t)\}}{\mathrm{FFT}\{u(t)\}}$. The frequency domain validation of the robust control design model is shown in figure 8. The

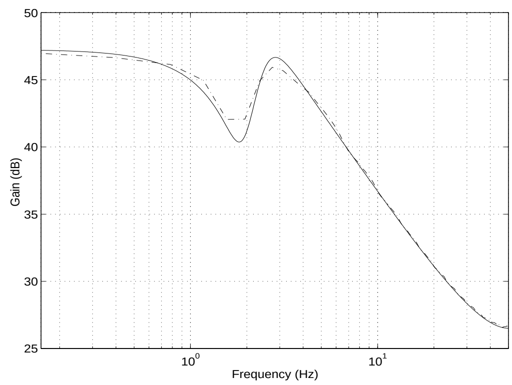

Fig. 8. Frequency response validation

identified model corresponding to 30 m/s on a dry road with the average mass has found to be the best robust control design model over the considered operating domain. Indeed, the velocity has shown to be relatively crucial for control system robustness considerations.

The design parameters have been specified to achieve the required specifications with adequate shapes of the usual sensitivity functions as shown in figure 9. The latter show the performance of the automatic steering system with the robust controller for its nominal situation (solid lines) and two other extreme situations corresponding to a speed of 15 m/s on snow road (dashed lines) and a speed of 5 m/s on a dry road (dotted lines). The corresponding modulus (MM) and delay margins (DM) as well as the attenuation bandwidths (AB) are shown in table 1.

Condition	MM(dB)	DM(sec)	AB(Hz)
nominal	-5.51	0.586	1.1
snow	-3.3	0.8	0.49
slow	-9.6	0.34	0.24

Table 1. Robustness margins and attenuation bandwidths

The design parameters involved in the control objective were $sh = 1$, $ch = 3$, $ph = 60$, $\rho = 30$, $W_u(z^{-1}) = \frac{1-0.4z^{-1}}{1+1.85z^{-1}+0.855z^{-2}}$ and $W_y(z^{-1}) = 1$. The predictor poles were set to 0.5, 0.6, 0.7, 0.8, 0.9 and 0.95. They have been obtained by damping the poles of the control model and adding one auxiliar pole.

Intensive simulations using the full-scale nonlinear model simulator have been carried out to emphasize the input-output behavior of the proposed automatic steering system. Four extreme driving conditions have been considered, namely

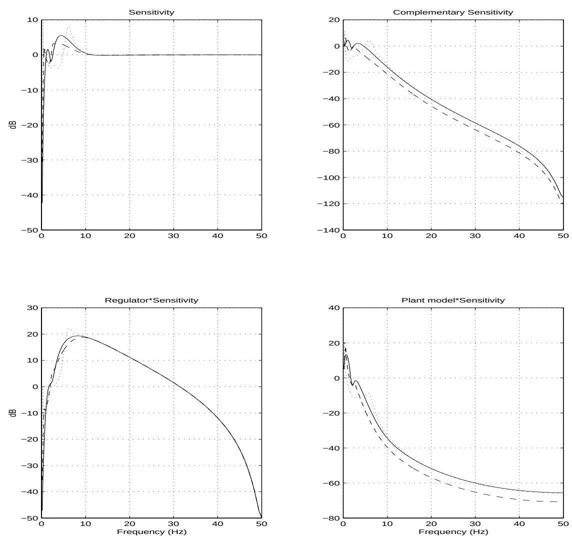

Fig. 9. Usual sensitivity functions

- a worst case corresponding to a speed of 30 m/s on a dry road (solid lines).
- a worst case corresponding to a speed of 18 m/s on a snow road (dashed lines).
- an average case corresponding to a speed of 15 m/s on a wet road (dash-dotted lines).
- an easy case corresponding to a speed of 5 m/s on a dry road (dotted lines).

Figures 10, 11, 12 and 13 show the corresponding lateral sensor displacements, steering angles, steering angle rates and yaw rates, respectively.

Notice that the actuator constraints as well as the passenger comfort specifications have been satisfied with the proposed robust controller. Recall that the yaw rate measurement has not been used in the feedback. However, it has been plotted as quantifier of the passenger comfort. Indeed, the yaw rate behavior has shown to be quite similar to that of the lateral acceleration that has been considered to be as a passenger comfort indicator in (Ackermann *et al.*, 1995).

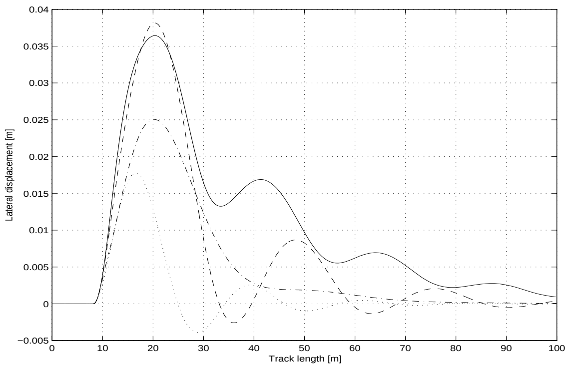

Fig. 10. Lateral displacements

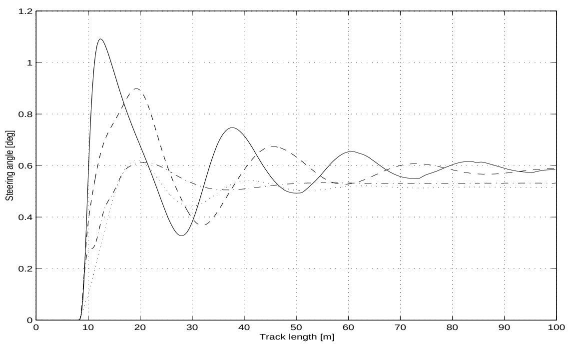

Fig. 11. Steering angles

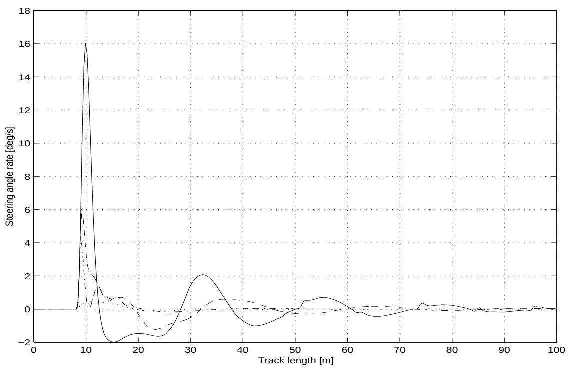

Fig. 12. Steering angle rates

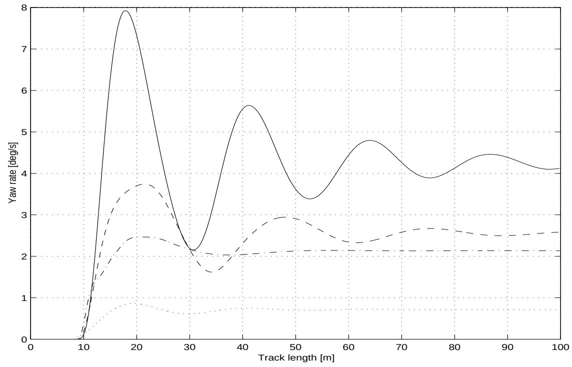

Fig. 13. Yaw rates

4. CONCLUSION

The motivation of this paper was to investigate the applicability of a robust predictive control approach for the design of an automatic steering system. The standard safety and passenger comfort specifications have been achieved. Three fundamental design features of the proposed control approach are worth to be pointed out, namely offset-free performance, stability robustness and implementation simplicity.

A great attention should however be paid to the identification of the control design model as well as the choice of design parameters. To this end, a comprehensive iterative procedure has been developed using the CACSD package SIMART. The latter has revealed to be a powerfull tool to deal with the advanced control design in quite realistic situations.

P.S. For the reference list, please contact the authors.

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