A Generic Multibody Vehicle Model for Simulating Handling and Braking

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SUMMARY

A vehicle system dynamics model is presented that captures the essential braking and handling behavior of an automobile with independent suspensions on a flat surface. The model, which has 18 degrees of freedom, is described in sufficient detail for an engineer to reproduce it with a multibody simulation program. Further, a stand-alone computer program based on the model has been put on the internet for interested readers to download and run. The model has a simple generic representation of suspension kinematics that can represent the behavior of most independent suspensions. The generic model is demonstrated using a previously published vehicle description (the IAVSD Iltis multibody benchmark) and compared with a detailed multibody model. Close agreement was found between the two models both for eigenvalues and for nonlinear time history responses.

1. INTRODUCTION

It is often said that an automobile is controlled by forces developed in just four small patches, each the size of a man's hand, where the tires contact the road. In the 1940's and 1950's, researchers such as Lanchester, Olley, Rieckert and Schunk, Rocard, and Segel developed an understanding of how tire forces are generated and affect the steering and braking behavior of the vehicle [1]. Segel and other early researchers in the 1950's developed linear equations by hand, and solved them using frequency-domain analysis [2]. Segel's classic model reduced the vehicle behavior to its essence, with a minimal number of parameters and variables and just three degrees of freedom (DOF).

From the 1960's to the early 1980's, the proliferation and improvement of analog and then digital computers led to a new phase of vehicle modeling, in which many automotive simulation programs were developed and refined by research engineers. The new computer models were more complex, typically with 10 to 20 DOF [3, 4]. The additional complexity accounted for nonlinearity and more detailed suspension kinematics. Equations were still formulated by hand, and coded by hand in computer language for numerical solution in specialized programs. The more detailed models involved many years of development, not counting the efforts spent in validation and verification.

Starting with the mid-1980's, engineers started using newly available multibody simulation programs to describe the model geometrically, "assembling" the system model from components [5, 6]. Modelers no longer had to derive equations, and therefore, the efforts and potential errors associated with deriving equations and coding them were nearly eliminated.

Automotive manufacturers and many others now use multibody programs to perform simulations of automotive handling and braking behavior [7]. The tendency has been to include nearly all moving parts in the suspensions and steering systems. Inputs include coordinates of most joints between parts, and mass properties of individual parts. In

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contrast, the earlier custom programs were more systems-oriented, involving generalized movements of wheels relative to the body, or, even more simply, movements of the body relative to the ground. The advantage of the detailed multibody programs for development engineers is that they can fine-tune designs by modifying component-level details. However, the detailed models also have some disadvantages. Engineers who do not work for car manufacturers may not have access to the geometric design data. Even when the full set of input parameters is assembled, the programs run slower than custom programs that are less complex. (With some multibody programs, the run-time performance is much slower even for comparable models. For models with a complexity similar to the one presented in this paper, a numerical multibody programs might be 50 times slower than a hand-written program specialized for a specific vehicle dynamics model.)

This paper is motivated by the thought that something has been lost during the evolution from the older models to the newer. The insight and expertise that underlay the old hand-written models are often lacking in modern multibody models. Although the modern models are often highly detailed, their accuracy in predicting vehicle response to steering and braking inputs is sometimes not as good as that obtained 40 years ago. This paper is intended to convey some of the ideas and concepts used in earlier vehicle models for applications using modern multibody programs.

A model will be presented that is intended for the engineer who has a multibody program and wants to simulate the essential braking and handling behavior of an automobile with full fidelity, without the overhead associated with component-level details (bushing rates, linkage geometry, etc.). The model that will be described was tested using the AUTOSIM multibody code generator [7, 8]. AUTOSIM generates equations symbolically, performs coding optimizations, and generates a custom simulation program. The simulation programs obtained by AUTOSIM have run-time performance comparable with (and usually better than) that of a hand-coded program based on the same model.

2. MULTIBODY INGREDIENTS

Readers who are familiar with the vehicle modeling literature may be disturbed to see less information than is typical for papers presenting models. This is because the model is intended to be implemented with a multibody program that will assemble the equations. Multibody programs handle details involving state variables, coordinate transformations, and equations of motion. The paper will focus on the physical significance of the modeling assumptions, and on auxiliary tire equations.

The major ingredients for describing the rigid-body kinematics and dynamics are bodies, points, and vector directions. Forces will be defined in terms of magnitudes, directions, and points that lie on the lines of action. The magnitudes and directions will usually be described in terms of quantities such as position and velocity vectors that are available from the multibody program with function such as `pos` (position vector) and `vel` (velocity vector). Some of the model degrees of freedom (DOF) are handled with auxiliary user-defined state variables and equations.

3. NOMENCLATURE

All of the symbols and unconventional functions that are used in this paper are listed below. Subscripts for wheels and suspensions are ommited when an equation applies to a single wheel. Vectors are shown in bold face characters. The nominal configuration of a multibody system in this paper represents the condition when all state variables are zero. Depending on the model, the nominal configuration may or may not be in equilibrium.

Symbols

α	delayed lateral slip angle (rad)
CTC	center of tire contact (point)
δ	steering of wheel relative to body (rad)
δ_s	steering due to steering system input from the driver (rad)
δ_{sw}	steering wheel angle (rad)
δ_k	steering due to suspension kinematics (rad)
δ_c	steering due to compliance in the suspension and steering system (rad)
Δ	suspension compression (also called jounce) (m)
F_s	compressive spring force, at spring (N)
F_x, F_y, F_z	components of tire force acting at point CTC (N)
φ_k	roll of wheel relative to body due to suspension kinematics (rad)
φ_c	roll of wheel relative to body due to compliance (rad)
γ	inclination (roll) of wheel relative to ground (rad)
H_{rc}	nominal roll center height (m)
H_{wc}	nominal height of wheel center (m)
Is	moment of inertia of wheel about spin axis (kg-m^2)
K_s	suspension spring rate (at spring) (N/m)
K_t	tire vertical spring rate (N/m)
κ	longitudinal slip ratio for a wheel (–)
L_{tk}	nominal track width (m)
L_{wb}	nominal wheelbase (m)
L_{relax}	tire relaxation length (m)
M_{by}, M_{dy}	moment about wheel spin axis due to brake or driveline (N-m)
M_f, M_r	mass of vehicle supported by front or rear tires (kg)
M_z	tire aligning moment (N-m)
θ	pitch of the sprung mass (rad)
$\mathbf{r}_z$	unit-vector perpendicular to road surface at tire contact point
R	instant rolling radius of tire (m)
R_{sg}	ratio of steering wheel angle divided by average road-wheel steer (–)
R_p	anti-pitch coefficient: ratio of longitudinal to vertical movement (–)
R_d, R_s	ratio of damper or spring compression to suspension compression (–)
$\mathbf{s}_x$, $\mathbf{s}_y$, $\mathbf{s}_z$	unit-vectors aligned with X, Y, Z axes of vehicle body
$\mathbf{t}_x$, $\mathbf{t}_y$	unit-vectors aligned with X, Y axes of tire/road contact
$\mathbf{t}_{x'}$, $\mathbf{t}_{y'}$	tire/road unit-vectors without effect of steer
τ	auxiliary state variable: tangent of delayed lateral slip angle (–)
$\mathbf{w}_y$, $\mathbf{w}_z$	unit-vectors aligned with Y, Z axes in wheel plane

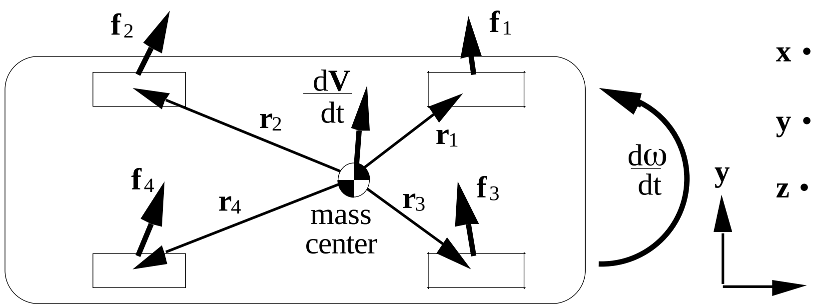
W_o	origin of wheel coordinate system, nominally at point CTC (point)
W_c	wheel center (point)
ω_s	auxiliary state variable: wheel spin rate (rad/sec)
v_x, v_y	scalar components of velocity vector of W_o (m/s)
X_c, Y_c, Z_c	coordinates of CTC in wheel body coordinate system (m)

Multibody Functions

dir(**r**)	direction of vector **r**: **r**/|**r**|
pos(P1, P2)	position vector going from point P2 to point P1
vel(P)	absolute velocity vector of point P

4. OVERVIEW OF FACTORS AFFECTING VEHICLE BEHAVIOR

Figure 1 shows a free-body diagram of a four-wheeled vehicle as viewed from the top. There are just three governing equations: the sum of the tire shear forces must equal the vehicle mass times its acceleration in both the vehicle X and Y directions, and the moment of those forces about the vehicle mass center must be equal to the product of the yaw acceleration and the vehicle yaw moment of inertia. Thus, the main objective of the vehicle model is to accurately predict tire shear forces.



$$\mathbf{x} \bullet \sum \mathbf{f}_i = \mathbf{x} \bullet M \frac{d\mathbf{V}}{dt}$$

$$\mathbf{y} \bullet \sum \mathbf{f}_i = \mathbf{y} \bullet M \frac{d\mathbf{V}}{dt}$$

$$\mathbf{z} \bullet \sum \mathbf{r}_i \times \mathbf{f}_i = \mathbf{z} \bullet I_{zz} \frac{d\omega}{dt}$$

Figure 1. Primary factors influencing vehicle system motions.

A vehicle is also subject to aligning moments in the tire contact patches. The aligning moment has a negligible direct effect on the vehicle yaw, but, due to steering compliance, it can be a significant factor in determining the all-important shear forces. Another behavior that influences the vehicle response involves the rotary motion of the car body in roll and pitch. Mechanical energy is transfered as the vehicle pitches and rolls, and these motions contribute to the vehicle transient response.

Besides the tire/road interactions, the only forces and moments acting on the vehicle are due to aerodynamic effects. They have a secondary influence, but are relatively easy to add to multibody models.

5. RIGID BODY KINEMATICS

The model is based on a rigid body that represents the main body of the vehicle and has six DOF. An additional four bodies are added, each with a single translational DOF, to account for the vertical movements allowed by the suspensions. The wheel bodies are positioned such that the origins of their local coordinate systems are nominally at the locations of the centers of tire contact (see Figure 2). Longitudinally, the origins of the front and rear wheels are separated by the vehicle wheelbase, L_{wb}. Laterally, they are separated by the vehicle front and rear track widths, $L_{tk,f}$ and $L_{tk,r}$.

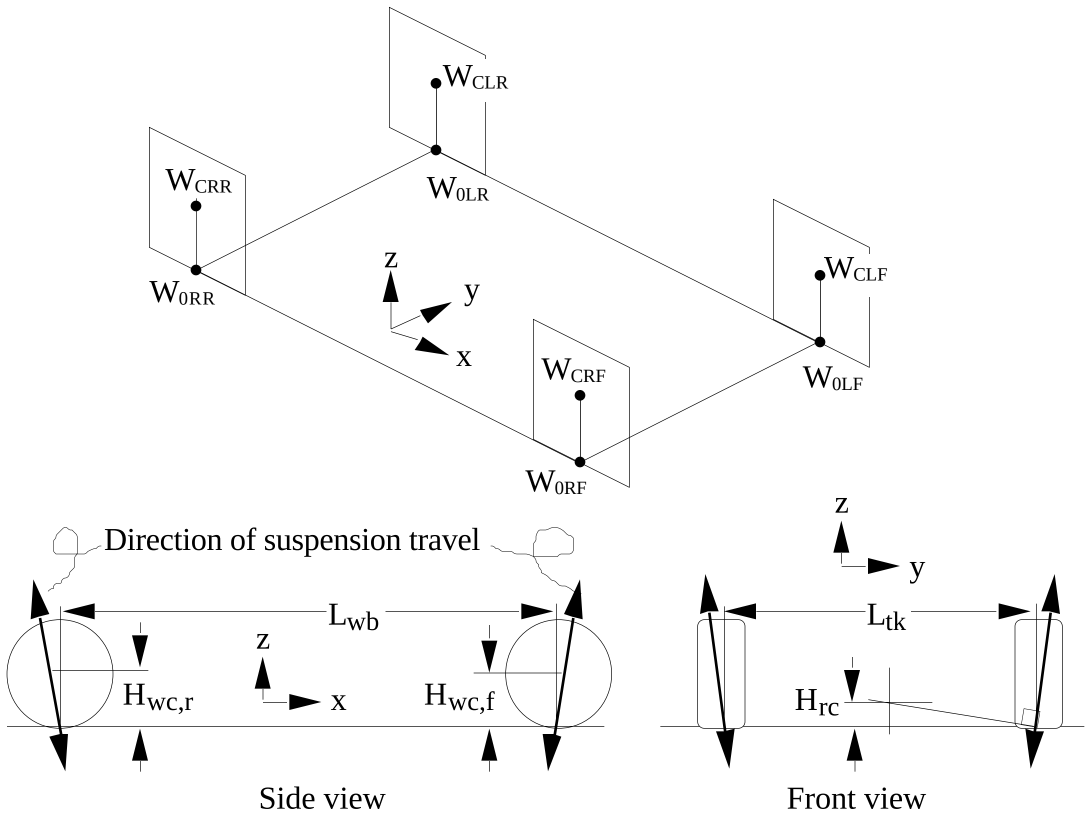


Figure 2. Locations and movements of wheels.

Ignoring, for the moment, the influence of compliances in the suspension and steering system linkages, each wheel center of a real vehicle follows a trajectory through 3D space, relative to the car body, as the suspension moves up and down. Due to the kinematics of the suspension, the trajectory is usually not purely vertical. For most vehicles, the wheels move out laterally as the suspensions are compressed, such that track width increases with suspension compression. The wheels also move out longitudinally, such that wheelbase increases with suspension compression.

The direction of the wheel trajectory (relative to the main body) determines how tire shear forces in the ground plane are transmitted to the vehicle body through reaction forces in the suspension linkages. In hand-written equations, roll and pitch moments due to suspension reaction forces have been written with coefficients with names such as anti-roll, anti-pitch, anti-dive, anti-squat, and jacking [3,4].

The multibody model accounts for the interaction between tire shear forces and roll and pitch moments so long as the movement is constrained to follow the proper path. A simple approximation is to assume the movement is in a straight line, as shown in Figure 2. Using an axis system based in the vehicle sprung mass ($\mathbf{s}_x$, $\mathbf{s}_y$, $\mathbf{s}_z$), the directions of the movements of the four wheels are:

$$\text{left-front: } \mathrm{dir}(\mathbf{s}_z + \frac{2H_{rc,f}}{L_{tk,f}}\mathbf{s}_y + R_{p,f}\,\mathbf{s}_x) \quad \text{right-front: } \mathrm{dir}(\mathbf{s}_z - \frac{2H_{rc,f}}{L_{tk,f}}\mathbf{s}_y + R_{p,f}\,\mathbf{s}_x)$$
$$\text{left-rear: } \mathrm{dir}(\mathbf{s}_z + \frac{2H_{rc,r}}{L_{tk,r}}\mathbf{s}_y - R_{p,r}\,\mathbf{s}_x) \quad \text{right-rear: } \mathrm{dir}(\mathbf{s}_z - \frac{2H_{rc,r}}{L_{tk,r}}\mathbf{s}_y - R_{p,r}\,\mathbf{s}_x) \quad (1)$$

where secondary subscripts f and r indicate parameters for the front and rear.

In the basic suspension analyses, the roll kinematics are analyzed to define a point called a roll center [9]. For compatibility with this convention, the inclination of the wheel movement in the roll direction is defined by the ratio of the roll center height to the half-track distance. For pitch, a single coefficient (R_p) is used.

6. MASSES AND INERTIAS

The user of a vehicle model must provide mass and inertia parameters for the bodies in the model. For the wheel bodies, one may set the moments of inertia to zero, and locate the mass centers at the wheel centers, nominally a height H_{wc} above the ground. The mass of each wheel body should be set to that portion of vehicle mass supported by the tire that is considered to move with the wheel. This value is commonly called the unsprung mass, and usually includes some of the mass of the suspension elements.

The mass of the main body, called the sprung mass, must be set to the mass of the entire vehicle minus the unsprung masses. The inertia properties are also required, including the XZ product of inertia. (Due to lateral symmetry, the XY and YZ products are usually zero.) It is much easier to measure inertia properties for the entire vehicle than for the body alone. The multibody program can be made to calculate the mass and inertia properties of the sprung mass from measurements made for the entire vehicle. This is done by adding four more bodies and giving them negative masses. These four bodies should be placed at the same locations as the wheel body mass centers. However, their masses should be set to the negative values of the unsprung masses, and they should be fully constrained with respect to the main vehicle body (i.e., zero DOF). The multibody program, in accounting for the full constraint of these four bodies, will in effect subtract the masses and inertia properties, bringing the mass and inertias of the main body down to those of the sprung mass alone.

7. SUSPENSION FORCE EFFECTS

Movement of a wheel along the line of motion allowed by the suspension kinematics is affected by suspension springs, dampers, bump stops, and anti-sway bars. In each case, some of the force generated by a component (e.g., a spring) acts to move the wheel, affecting the transfer of mechanical energy to and from the sprung mass. In addition, some of the force is reacted at other points or in other directions that do not move and therefore cannot affect the transfer of mechanical energy. For example, consider the spring shown in the suspension of Figure 3. If the wheel moves vertically an amount of Δ relative to the body, the spring is compressed by a lesser amount, say, for example, $R_s\ \Delta$, where R_s is a coefficient that defines the mechanical advantage of the spring relative to the wheel. The spring exerts a force F_s on the lower control arm. Some of the force is reacted at the

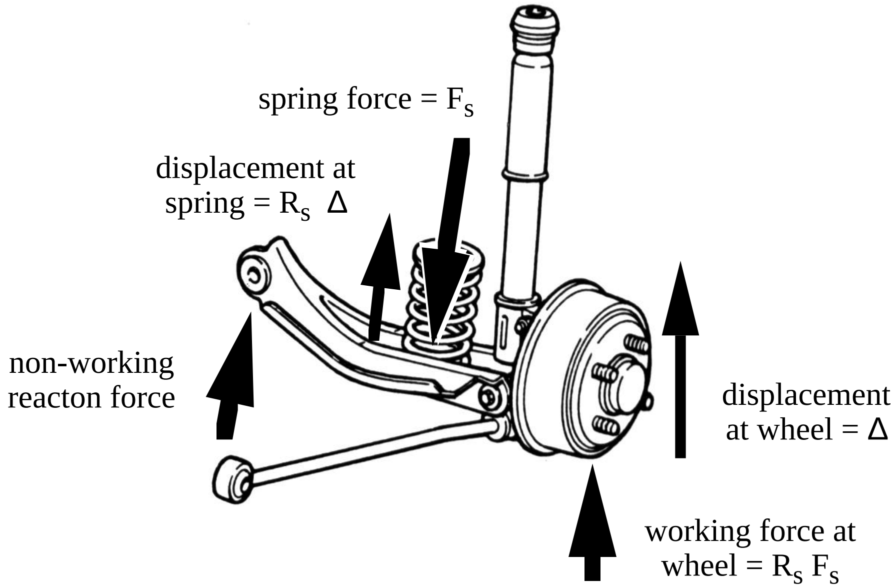


Figure 3. Mechanical advantage of suspension component.

connection to the body, and some is reacted at the wheel by the vertical tire force, as shown in the figure. Conservation of work requires that the change in force at the wheel center multiplied by its movement must be equal to the change in spring force, multiplied by its change in compression. Thus, the effect of the spring at the wheel is R_s F_s. A similar analysis can be made for the damper, using a different ratio R_d.

This principle of mechanical advantage is used to include components such as springs and dampers in the vehicle dynamics model without directly modeling their points of attachment or the complex suspension linkage geometry. The effect of a suspension component at the wheel is calculated in three steps:

1. multiply the suspension compression (measured at the wheel) by the kinematic ratio to determine the compression at the component,
2. apply a known functional relationship (e.g., spring force vs. compression) to determine the force generated by the component, and
3. multiply the component force by the kinematic ratio to obtain an effective vertical force at the wheel.

For a linear spring, the three steps can be combined to define an effective spring rate at the wheel: K_s R_s^2. For nonlinear relations, it is necessary to perform all three steps. For example, consider the addition of the suspension spring for the left-front wheel. In AUTOSIM, the force is added with a command called `add-line-force`, with arguments shown in Table 1 for the spring force.

A similar treatment is made for the shock absorber, using the derivative of the suspension displacement, the damper mechanical advantage R_d, and a functional relation between damper force and stroke rate. In general, different geometric ratios are needed for the suspension spring, the damper, and the bump stop. Different ratios are also used for front and rear, but the same ratios are used between left and right wheel on the same axle.

Table 1. Adding a spring force to the model.

Property	Value	Description
Point 1	P1: Point on sprung mass	Point on which force acts
Point 2	P2: Point on wheel	Point in body taking the reaction force
Direction	$\mathbf{s}_Z$	Direction of force vector
Magnitude	$R_S\ F_S(R_S\ \mathbf{s}_Z \bullet \text{pos}(P1, P2))$	Magnitude of force vector

The effect of the anti-sway bar is modeled with a linear spring between the two wheels linked by the bar. The two points in the `add-line-force` command are on the two wheels, the direction of the force is $\mathbf{s}_Z$, and the magnitude is a spring rate multiplied by the vertical movement difference between the two points.

8. TIRES

The primary challenge in developing a valid vehicle simulation model is to accurately predict the tire forces. Although the multibody program handles the kinematics and dynamics of the rigid bodies in the system, it is usually necessary to use an external (user-defined) routine to compute tires forces and moments based on kinematic inputs. Many tire models (algorithms) exist for calculating tire forces [10], and most require the same inputs: vertical load (F_Z), longitudinal slip (κ), lateral slip (α), and inclination angle (γ). (Some models also require surface friction and wheel speed.) As outputs, they calculate longitudinal force (F_X), lateral force (F_Y), and aligning moment (M_Z).

To properly determine the tire forces and factor them into the vehicle model, it is necessary to (1) define a point where the tire forces act on the multibody model; (2) determine an expression for the vertical tire force F_Z that is required as an input for the tire model; (3) establish the vector directions for the X and Y components of the tire shear force relative to the vehicle model; (4) determine expressions for the kinematical inputs required by most tire models (longitudinal slip, lateral slip, and inclination angle); and (5) use a tire model to determine the magnitudes of F_X, F_Y, and M_Z.

Tire Relaxation Length

Tires develop shear forces in response to deformation of the tire structure. The forces do not develop instantly, but build as the tire rolls. Researchers have found that the dynamic delay of the forces is primarily linked to the spatial distance covered by the tire [11]. The earliest approximation of this behavior was to treat the delay as a first-order lag. The characteristic parameter is called relaxation length, and is similar to a time constant, except that it has units of length rather than time. The response to steering is delayed sufficiently that the lag interacts with the dynamics of the vehicle system at low speed [12]. The lag for longitudinal slip is usually neglected.

Two methods are commonly used for including the tire lag in a vehicle model: (1) use a tire model with the dynamics built in, or (2) use a static (steady state) tire model with a separate filter to account for the lag. The second approach is preferable because it offers two practical advantages. First, it allows us to use any static tire model from the literature, independently of the method used to introduce lag. Second, it simplifies the calculation of the kinematical variables used as inputs to the tire model. Thus, we will introduce lag into the slip angle, such that the instant response calculated for the lagged slip angle yields the lagged side force and aligning moment.

Center of Tire Contact

The point of application for the three components of the tire force is called the center of tire contact (CTC). As the wheel moves up and down, point CTC remains in the ground plane (see Figure 4). CTC lies on the (negative) Z axis of a wheel coordinate system whose Y axis is the wheel's spin axis and whose origin is the wheel center. The distance between the wheel center W_c and CTC is the instant radius R:

$$R = \frac{\mathbf{r}_z \bullet \mathrm{pos}(W_c)}{\mathbf{w}_z \bullet \mathbf{r}_z} \tag{2}$$

where the function $\mathrm{pos}(W_c)$ defines a position vector going from any point fixed in the ground plane to the wheel center.

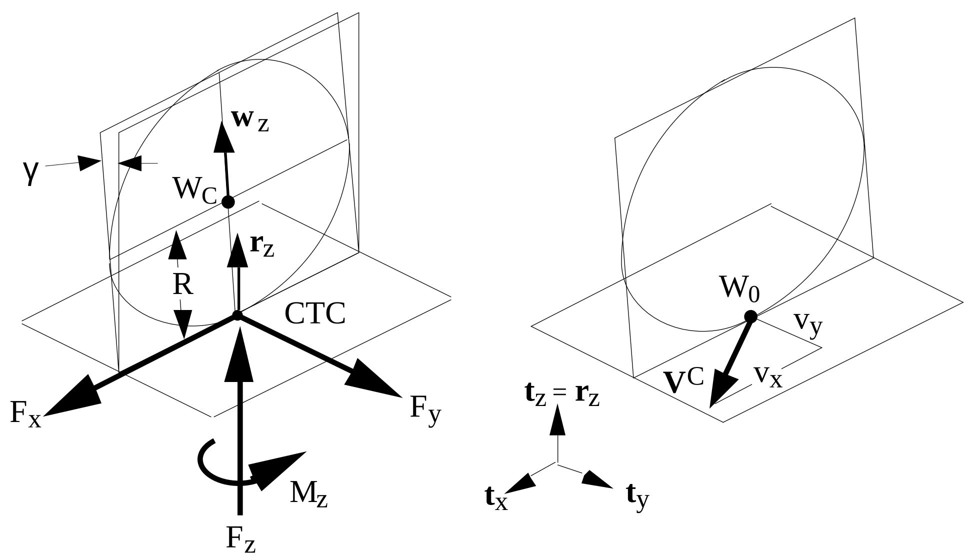


Figure 4. Tire points and axes.

Due to the simplified suspension kinematics described earlier, the rigid bodies representing the wheels do not steer or incline relative to the main vehicle body. Thus, they have the same roll and pitch angles relative to the ground as the main body. The coordinate system origin is such that the coordinates of point CTC are nominally zero. However, for an arbitrary vehicle configuration, the coordinates are:

$$X_c = \theta\, R \qquad\qquad Y_c = 0 \qquad\qquad Z_c = H_{wc} - R \tag{3}$$

Vertical Tire Force

The vertical tire load for a wheel is defined with the expression

$$F_z = \max(1.0, \frac{M_i\ g}{2} - K_t\, Z_c) \tag{4}$$

where the index in M_i indicates either the mass supported by the front or rear tires. This definition of F_z establishes the force to be the equilibrium value M_i g/2 when the vehicle is in the nominal configuration. The use of the max function prevents the magnitude of the force from going below 1.0 N. The lower limit of 1.0 N is used rather than the actual limit of 0 N to avoid numerical problems with some tire models.

Tire Axes

The tire X and Y axes, lie in the plane of the road and are parallel to unit-vectors $\mathbf{t}_x$ and $\mathbf{t}_y$ shown in Figure 4. In our multibody model, we do not have the true wheel orientation because of the simplified suspension kinematics. Thus, the tire X and Y axis directions are defined as:

$$\mathbf{t}_{x'} = \mathbf{s}_y \times \mathbf{r}_z \qquad \mathbf{t}_{y'} = \mathbf{r}_z \times \mathbf{t}_{x'} \tag{5}$$

$$\mathbf{t}_x = \cos(\delta)\,\mathbf{t}_{x'} + \sin(\delta)\,\mathbf{t}_{y'} \qquad \mathbf{t}_y = \cos(\delta)\,\mathbf{t}_{y'} - \sin(\delta)\,\mathbf{t}_{x'} \tag{6}$$

where δ, the steer angle of the wheel plane relative to the vehicle body, is defined as:

$$\delta = \delta_s + \delta_k + \delta_c \tag{7}$$

The first term, δ_s, accounts for the input of the driver and is typically defined with an equation such as the following:

$$\delta_s = \mathrm{str}\left(\frac{\delta_{sw}}{R_{sg}}\right) \tag{8}$$

where str is a table-lookup function that calculates steering angle as a nonlinear function of geared-down steering input. Different functions would typically be used for the left and right sides, to account for Ackerman geometry. The second term in eq. 7, δ_k, sometimes called bump steer, accounts for steer due to suspension kinematics. It would typically be calculated using a nonlinear table-function relating toe to suspension compression. Toe is defined as positive with the wheels steering inward. Therefore, δ_K equals toe for the wheels on the right-hand side of the vehicle, and δ_k equals negative toe on the left side. The third term in eq. 7, δ_c, accounts for compliance in the suspension and steering system. It is typically calculated using measured linear coefficients that define the amount of steer caused by unit changes in F_x, F_y, and M_z.

Kinematical Inputs to Tire Model

The slip quantities required by a tire model are based on the velocity of point CTC. In nearly all vehicle dynamics models that have been developed by hand (without multibody programs), approximations have been used because the exact expressions for the CTC velocity can be complicated. With a multibody program such as AUTOSIM, the exact expressions can be obtained with little effort on the part of the modeler. However, the complexity of the exact equations results in longer equations, and slower simulation speed. Most of the complexity comes from terms that are infinitesimal and negligible. The wheel bodies are defined such that the origin of the wheel coordinate system (W_o) nominally coincides with CTC. To obtain shorter run times, the point W_o, fixed in the wheel body, is used to define tire velocity rather than the point CTC. The local components of speed are:

$$v_x = vel(W_o) \bullet \mathbf{t}_x \qquad v_y = vel(W_o) \bullet \mathbf{t}_y \tag{9}$$

Longitudinal slip (κ) is defined for each wheel by the equation:

$$\kappa = \frac{\omega_s R - v_x}{v_x} \tag{10}$$

Under this definition of longitudinal slip, κ goes from –1 at full braking lock-up, to zero for the free-rolling condition, to +∞ for spinning the wheel when the vehicle is at rest or moving sideways ($v_x = 0$). To avoid numerical problems, κ can be calculated with a user-defined function that checks for conditions of v_x approaching zero.

The wheel spin, ω_s, is an auxiliary state variable whose derivative is defined as:

$$\frac{d\omega_s}{dt} = \frac{M_{dy} - M_{by}(p_{in}) \frac{\omega_s}{|\omega_s|} - R\ F_x}{I_s} \tag{11}$$

(In AUTOSIM, the commands `add-state-variable` and `add-equation` are used to insert auxiliary equations such as eq. 11.) The sign of ω_s is used to ensure that the braking torque (M_{by}) always opposes the wheel spin. The brake torque itself is a table-lookup function (M_{by}) of a variable, p_{in}, that is the input from the brake pedal. To avoid a divide-by-zero error, the computer function `sign` is actually used in the model, rather than the ratio of $\omega_s/|\omega_s|$.

Lateral slip is defined as the arctangent of an auxiliary state variable, τ.

$$\alpha = \tan^{-1}(\tau) \tag{12}$$

Recall that tire relaxation length adds a lag to the lateral slip angle. A method described by Bernard [13] is used, in which a state variable is added for each wheel and defined with a first-order differential equation:

$$\frac{d\tau}{dt} = \frac{|v_x|}{L_{relax}} \left[\frac{v_y}{|v_x|} - \tau \right] = \frac{1}{L_{relax}} [v_y - |v_x| \tau] \tag{13}$$

The absolute value of v_x is used in eq. 13 to maintain continuity in case the vehicle spins out and v_x assumes a negative value.

Inclination of the wheel plane relative to the ground is the sum of the roll (relative to the ground at point CTC) and the roll of the wheel relative to the vehicle:

$$\gamma = \sin^{-1}(\mathbf{s}_y \bullet \mathbf{r}_z) + \varphi_k + \varphi_c \tag{14}$$

The kinematical term, φ_k, is typically a table-lookup function of camber vs. suspension compression for each axle. Camber is defined as positive when the wheels lean out at the top. Thus, for wheels on the right-hand side, φ_k equals camber, and on the left side, φ_k equals negative camber. The compliance term, φ_c, can usually be neglected. If not, it is defined by a linear compliance coefficient that relates change in wheel inclination to a unit change in F_y.

Sequence of Calculations

The vehicle simulation is run by numerically integrating a set of ordinary differential equations. The model described above has 18 degrees of freedom (DOF): ten for the five rigid bodies (six for the main body, and one for each wheel), and the other eight for the auxiliary state varialbes ω_s and τ that are added for each wheel. At the beginning of each time step, the values of all of the state variables are known, including the ω_s and τ variables. The tire factors for the four wheels are then calculated in the following sequence:

1. Compute terms that depend only on the state variables: F_z, γ, and κ. If camber compliance is included, use the tire forces from the previous time step to compute γ.
2. Compute F_x, F_y, and M_z with a static tire model.
3. Compute wheel steer, including the effect of steer compliances coupled with tire actions F_x, F_y, and M_z.
4. Determine the directions of the tire X and Y axes.
5. Calculate terms that depend on knowledge of the directions of the tire axes:
    a. Apply forces F_x and F_y to the multibody model at the point CTC.
    b. Calculate derivatives of ω_s and τ.

9. EXAMPLE

The multibody model described in this paper was compared with a full multibody model for the Iltis Bombardier benchmark vehicle description contained in a special issue of *Vehicle System Dynamics* [7]. The Iltis model has the equivalent of short-long arm (SLA) suspensions at all four wheels, with a tie rod to control steer angle. Neglecting wheel spin, the system has 18 moving parts, and ten dynamical DOF. There are three springs for each wheel: a leaf spring with a linear spring rate, a nonlinear spring within the shock absorber, and a bump stop. The damping is nonlinear. The tire forces and moments are calculated with the Calspan tire model. As part of the preparation of the special issue, the full Iltis equations of motion were generated by using AUTOSIM. Table 2 compares the 18-DOF model and the full multibody model in terms of model complexity and simulation speed.

Table 3 compares the eigenvalues for five models. The mode descriptions and the Medyna and Simpack eigenvalues are taken from the published benchmark description [7]. The values in the AUTOSIM column are from the full Iltis model, and the other two columns are for the 18-DOF model, with and without the effect of tire relaxation length ($L_{relax} \approx 0$, $L_{relax} = 0.6$ m). The eigenvalues were calculated for the case of the vehicle in equilibrium, running at 20 m/s. Although slight differences can be seen due to tire relaxation, the differences are small at this speed. (The effect becomes more significant at lower speeds.)

Table 2. Comparison of model complexity.

	Full	18-DOF
Number of moving bodies handled by AUTOSIM	18	5
Number of auxiliary state variables	0	8
Total number of DOF	10	18
Suspension kinematics	Full	Simple
Mechanical advantage of springs and dampers	Exact	Linear ratio
Change in steer, camber due to suspension compression	Exact	Table
Tire model	Calspan	Calspan
Tire relaxation delay, wheel spin dynamics	No	Yes
Nonlinear spring, damper	Exact	Table
Suspension/steering compliance	None	Linear
Number of multiplies and divides	6847	1081
Number of adds and subtracts	4951	1033
Number of function calls	115	80
Time step for normal handling	0.005	0.005
Ratio: Computation time / Simulated time (on HP 715)	2.12	0.25

Table 3. Comparison of Eigenvalues for five versions of the Iltis benchmark.

Mode	Medyna	Simpack	AUTOSIM	18-DOF[1]	18-DOF[2]
veh. long.	±0.000[3]	-0.000±0.000i	0	0	0
veh. lateral	+0.003	+0.000	0	0	0
veh. lateral	0	-0.000	0	0	0
cabin roll	–17.7	–16.8	–17.5	–17.9	–14.6
cabin roll	–40.4	–41.0	–37.6	–33.9	–28.0
lat., roll	–3.61 ± 4.40i	–3.95 ± 4.10i	–3.72 ± 4.15i	–3.62 ± 4.31i	–3.13 ± 4.74i
pitch	–4.00 ± 9.30i	–3.96 ± 9.47i	–3.98 ± 9.50i	–4.01 ± 9.15i	–4.10 ± 9.22i
bounce	–4.86 ± 11.1i	–4.92 ± 11.1i	–4.94 ± 11.1i	–4.83 ± 11.2i	–4.79 ± 11.2i
hop/roll	–27.0 ± 73.4i	–26.8 ± 73.4i	–28.8 ± 73.1i	-27.0 ± 73.8i	–25.7 ± 70.9i
hop/bounce	–28.8 ±82.4i	–29.9 ± 81.9i	–30.2 ± 82.0i	–28.4 ± 80.5i	–27.8 ± 81.6i
hop/pitch	–30.8 ± 82.3i	–29.8 ± 82.8i	–30.1 ± 82.9i	–28.6 ± 81.6i	–27.8 ± 82.5i
hop/yaw	–29.3 ± 85.9i	–29.4 ± 85.9i	–29.7 ± 86.1i	-28.5 ± 84.5i	–27.8 ± 85.1i

[1] 18-DOF model with $L_{relax} \approx 0$; [2] 18-DOF model with $L_{relax} = 0.6$ m

[3] numbers whose magnitude are > 0 and < 0.0005 are shown as ±0.000

Figure 5 compares time histories for a step-steer input at three vehicle speeds, as described in the benchmark. The runs start at -3 seconds with the vehicle in its nominal configuration. The first plot shows how both models bounce a little as they go into static equilibrium. The other three plots cover the range of 0 to 5 seconds. The lateralacceleration and yaw rate plots show close agreement between the detailed and 18-DOF models, but the

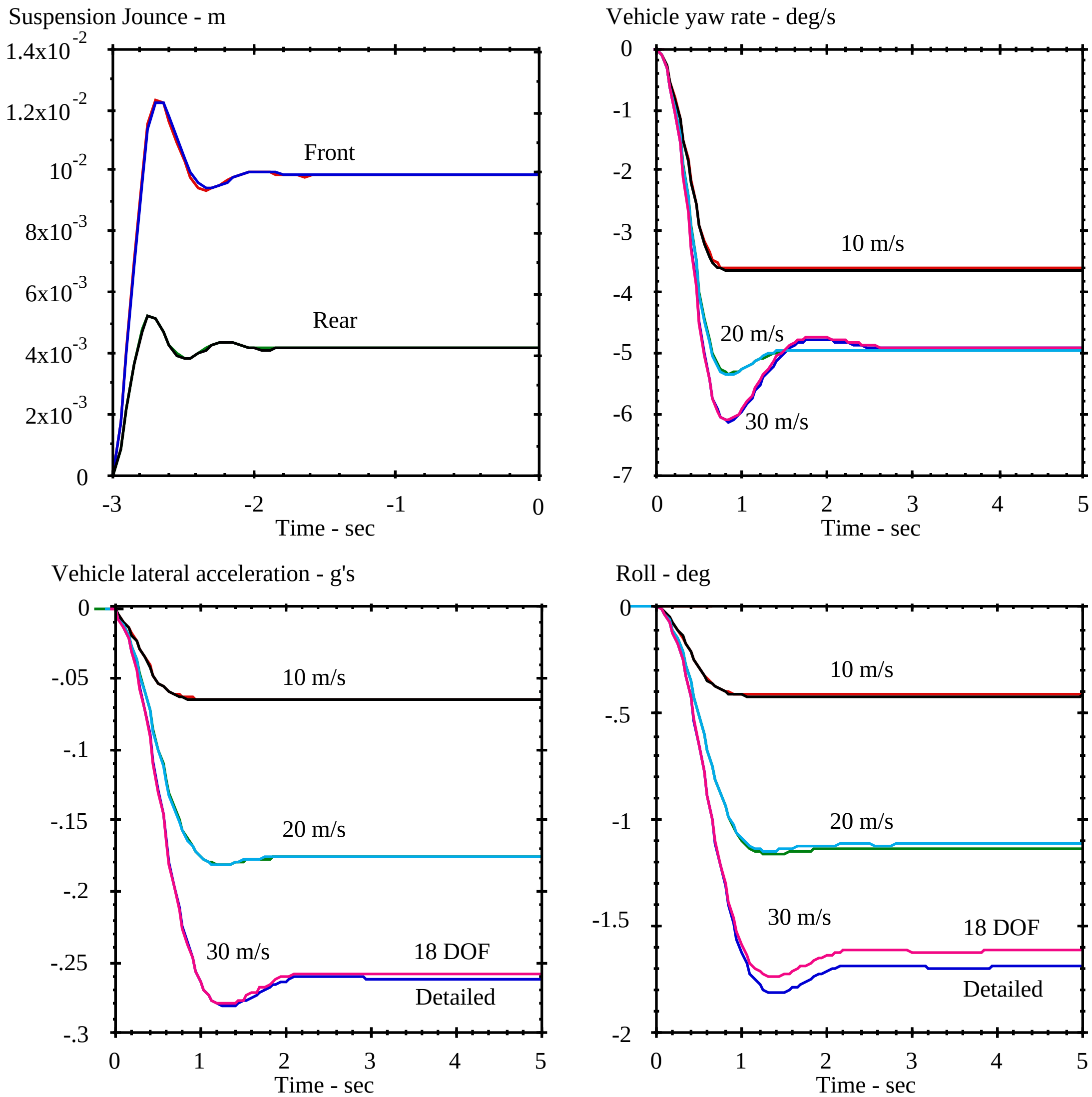


Figure 5. Time histories from two models of Iltis.

plot of roll angles shows a difference of 0.1° at the steady state. This is perhaps due to the difference between the full kinematical representation of the suspension in the detailed model and the simplified one in the 18-DOF model.

Computer software with executable programs for PC/Windows and Macintosh computers is available for free download from the internet. In addition to simulation programs generated by AUTOSIM, there is a simulation graphical user interface (SGUI) [14] to simplify the viewing and changing of vehicle parameters. There are also plotting and animation tools for viewing simulation results. Access the main WWW link `html://www.umtri.umich.edu/` and follow the links to obtain the software. If any problem should be found downloading the software, please contact the authors for help (E-mail: `autosim@umich.edu`, `msayers@umich.edu`, and `donhan@umich.edu`).

10. CLOSURE

An 18-DOF model was presented that is intended to capture the essential braking and handling behavior of an automobile with independent suspensions on a flat surface. It was described for the engineer who has a multibody program and wants a working vehicle dynamics model. Unlike many other vehicle multibody models, this has a simple generic representation of suspension kinematics that can represent almost any kind of independent suspension. A known benchmark vehicle model was run with the 18-DOF model and shown to provide nearly identical results in comparison with the fully detailed multibody description.

Although the Iltis model description provides a valuable resource for verifying new computer models, it lacks two important features of real vehicles: (1) steering compliance (due to bushings and bending of metal), and (2) tire relaxation length.

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