Cylinder Pressure in a Spark-Ignition Engine: A Computational Model

PAULINA S. KUO

The project described in this article attempts to accurately predict the gas pressure changes within the cylinder of a spark-ignition engine using thermodynamic principles. The model takes into account the intake, compression, combustion, expansion and exhaust processes that occur in the cylinder. Comparisons with actual pressure data show the model to have a high degree of accuracy. The model is further evaluated on its ability to predict the angle of spark firing and burn duration.

Introduction

Gas pressure in the cylinder of an engine varies throughout the Otto four-stroke engine cycle (see ref. 1 for an elaboration of this process). Work is done on the gases by the piston during compression and the gases produce energy through the combustion process. These changes in energy combined with changes in the volume of the cylinder lead to fluctuations in gas pressure. The ability to accurately predict the pressure allows for better understanding of the processes taking place in the cylinder such as the interactions between the gases, oil film, piston and liner. In the first phase of this project, a computer model was developed to predict pressure based on initial conditions and engine geometries.

A problem encountered in gathering data was the fact that the spark timings were not accurately known due to variations in engine speed. The exact moments of spark firing and cessation of combustion were unknown. Spark firing was estimated to be at 25˚ to 5˚ before top center (BTC) and the burn duration is approximately 60˚ to 80˚. The second phase involved determining the most precise values for spark firing and burn duration by comparing the model to actual data.

The Model

The model, which was programmed in FORTRAN, predicts the cylinder pressure throughout the intake, compression, combustion, expansion and exhaust processes that make up the engine operating cycle. Pressure was modeled as a function of the angle of the crank (see ref. 1), which ran for 720 degrees per cycle or two revolutions because the crank completed two rotations per cycle. The valve and spark timings, engine geometry, engine speed and inlet pressure were entered into the model.

The individual processes of the engine cycle, intake, compression, combustion, expansion and blowdown/exhaust, are discussed below in order of occurrence.

Intake. Intake occurs between exhaust valve closing (EVC) and the start of compression. The intake valve opens before the exhaust valve closes, so there is a period of overlap during which both valves are open. During this period, the model used an s-curve to describe the gradual transition between exhaust pressure and intake or inlet pressure. The start of compression, which marks the end of the intake process, does not necessarily occur at the same time as the close of the intake valve (IVC). The intake valve closes after bottom center (BC) while the volume in the cylinder is decreasing. The engine speed determines the point at which the fuel/air mixture stops flowing into the cylinder. At lower revolutions per minute (rpm), the start of compression is closer to IVC; at higher speeds, it is closer to BC. An s-curve approximates the angle of the start of compression as a function of engine speed.

The volume of the cylinder during intake increases as the piston descends, thereby drawing in the fuel mixture. There is little resistance to gas flow into the cylinder, which causes the pressure in the cylinder to remain relatively constant and equal to the inlet pressure.

Compression. Both the intake and exhaust valves are closed during the compression stroke so that the gases can neither enter nor exit the cylinder. The piston is moving upward, so cylinder volume decreases. Pressure increases as the gas in the cylinder is compressed. Because of the high speed of the piston, the duration of compression is short and negligible heat is lost to the walls of the cylinder. Relatively little energy is dissipated due to internal friction of the gas. Overall, there is little change in entropy during compression, and the gas behavior can be described by the equation:[2]

$$PV^{\gamma} = constant. \tag{1}$$

This equation allowed for calculation of the cylinder pressure at any crank angle during compression based on the knowledge of initial pressure and volume, P_0 and V_0, which determine the constant. The volume of the cylinder is a direct function of crank angle, cylinder geometries, crank radius and connecting rod length (see ref. 1). The ratio of the specific heat of the fuel at constant pressure to the specific heat at constant volume is γ; its value varies from compression to combustion to expansion. During compression, γ is approximately equal to 1.3.[3]

Combustion. The combustion process was described by the McCuiston, Lavoie and Kauffman (MLK) model.[4] The mass-burn fraction, χ_b, was modeled as a function of pressures and volumes:

$$\chi_b = \frac{PV^n - P_0V_0^n}{P_fV_f^n - P_0V_0^n} \tag{2}$$

where P = pressure corresponding to burn fraction
V = volume corresponding to burn fraction
P_0 = pressure at start of combustion
V_0 = volume at start of combustion
P_f = pressure at end of combustion
V_f = volume at end of combustion
n = polytropic constant

PAULINA S. KUO is a recent graduate of Thomas Jefferson High School for Science and Technology. She completed the research described in this article under the supervision of Professor John Heywood at the MIT Sloan Automotive Laboratory. Ms. Kuo was honored as a Westinghouse Science Talent Search finalist for this work, and she was a member of the 1996 US Physics Team; she aspires to a career as an engineer or a professor.

The polytropic constant, n, is an empirically determined constant about equal to γ which has a value close to 1.25 during combustion.[3]

The fraction of fuel burned varies with an s-curve which runs from 0% burned at the start of combustion to 100% burned at the end of combustion. The Weibe function is an s-curve that is used to describe the burn fraction:[1]

$$\chi_b = 1 - \exp\left[-a\left(\frac{\theta - \theta_0}{\Delta\theta}\right)^{m+1}\right] \tag{3}$$

θ_0 is the angle at which combustion begins; it is about equal to the angle of spark firing. The angle of the duration of combustion is $\Delta\theta$. The constants a and m are determined experimentally. Real burn fraction curves have been fitted by the Weibe function with a=5 and m=2.

The terms in equation (2) can be rearranged to give an expression for pressure in terms of the crank angle and the conditions at the onset and end of combustion. The final conditions can be represented as functions of the initial conditions by using the fact that the gases in the cylinder act almost ideal and that negligible energy is lost or gained by the system. Since the combustion process occurs almost symmetrically about top center (TC), the volume of the cylinder at the spark is approximately equal to the volume at the end of combustion. Because the initial and final volumes are about equal, little net work is done on the piston and the change in temperature between the spark and end of combustion is due to the burning of the fuel.

According to the ideal gas law, the gas pressure at the end of combustion, P_f, is a function of the gas constant of the fuel, mass of the gases, temperature, and volume. The gas constant of the fuel is equal to the universal gas constant divided by the molar mass of the fuel. The total mass of the gas mixture was calculated at IVC, just when the cylinder is sealed, by using the ideal gas law. Temperature at the end of combustion was estimated using the assumption that nearly all of the chemical energy of the fuel caused a temperature change of the gases. This idea is expressed in the following equation:[5]

$$m_{total}\, c_v (T_f - T_{spark}) = C\, m_{fuel}\, Q_{HV} \tag{4}$$

where m_{total} = total mass of gases in the cylinder
c_v = specific heat of gas mixture at constant vol.
T_f = temperature at the end of combustion
T_{spark} = temperature at the start of combustion
C = coefficient for unburned fuel
m_{fuel} = mass of fuel in the cylinder
Q_{HV} = heating value of the fuel

The temperature at the spark was calculated by applying the ideal gas law to the conditions at the end of compression. The constant C took into account the fact that not all of the fuel was burned and thus not all of the chemical energy available was changed into thermal energy; C is approximately 0.95. The mass of the fuel was calculated by using the air-to-fuel ratio and noting that the total mass is equal to the mass of air plus the mass of fuel. The heating value is a constant dependent upon the type of fuel used. For gasoline, the heating value is about 44 megajoules per kilogram.

Expansion. The end of combustion occurs slightly after TC at which point expansion begins. The pressure of the burned gases drives the piston down, does work by turning the crankshaft and, in turn, provides power to the car. During expansion, the heat transferred to the cylinder liner is small compared to the work done. Energy lost to internal friction of the gas is also minimal. The gas behavior was described by equation (1) which is also used to model the compression process. The conditions at the end of combustion determined the constant term. During expansion, γ has a value of about 1.48.[3]

Blowdown and Exhaust. Exhaust valve opening (EVO) occurs before the crank reaches BC. At this point, the pressure in the cylinder is much greater than the exhaust system pressure. The higher pressure in the cylinder helps push the burnt gases out of the cylinder. The process by which the pressure aids in the expulsion of burnt gases is called blowdown. The flow of the gases can be described by a model of gas flow through an orifice where the valve acts like the flow restriction.[1] This model depends on the velocity of the gas. When the gas velocity at the smallest portion of the opening, the throat, is equal to the speed of sound, the flow is said to be choked. The flow rate under choked flow is described by:

$$\dot{m}_{real} = \frac{C_D A_T p_0}{\sqrt{R T_0}} \gamma^{1/2} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5}$$

where $\dot{m}_{real}$ = real mass flow rate
C_D = discharge coefficient
A_T = cross-sectional area of throat
p_T = pressure at throat
p_0 = pressure in the cylinder
T_0 = temperature in the cylinder
R = characteristic gas constant of the exiting gases
γ = ratio of specific heats of the exiting gases

Subsonic flow is described by the equation:

$$\dot{m}_{real} = \frac{C_D A_T p_0}{\sqrt{R T_0}} \left(\frac{p_T}{p_0}\right)^{\frac{1}{\gamma}} \left\{\frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{p_T}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \tag{6}$$

The discharge coefficient is an experimentally determined constant which relates the effective throat area to the actual throat area. It is roughly equal to 0.7. Secondary flow near the throat cause the main flow to pass through a smaller area than the actual cross-sectional area of the throat. The pressure at the throat is equal to that of the gases in the exhaust system. $\dot{m}$, p_0 and T_0 are variable but can be related by the ideal gas law:

$$P_0 V = m R T_0 \tag{7}$$

and the isentropic process gas model:[2]

$$\frac{T_0}{P_0^{(\gamma-1)/\gamma}} = constant \tag{8}$$

These models are valid because there is negligible entropy change in the system and the gases in the cylinder do not deviate far from ideal behavior.

The resultant equation is a differential equation where the mass flow rate, $\dot{m}$, is dependent on the mass of the gases in the cylinder, m. The value of m is computed numerically by using the three-step Runge-Kutta algorithm and then used to find the gas pressure.

The flow of the gases depends on the area of the opening of the exhaust valve. This area changes as the valve is pushed open and closed. It increases quickly to a maximum, reaches a plateau and falls off again as the valve closes. The maximum area is determined mainly by the shape of the duct to the exhaust system. Pressure in the cylinder settles down to the exhaust system pressure as the exhaust valve remains open. The valve closes after TC after the next engine cycle has begun.

The exhaust system pressure is a function of engine speed. There are several barriers to the exiting gas in the exhaust system, such as the catalytic converter and the muffler, which resist the flow of the burnt gases and cause the pressure in the exhaust system to increase roughly as the square of the speed of the exiting gases. The speed of the gases is directly proportional to the engine speed. Generally, the exhaust pressure is between 1 atmosphere (atm) and 1.5 atm.

Model Evaluation

Data was collected from a Renault production engine at different engine speeds and inlet pressures. The geometries of the engine were entered into the model and the model was then used to predict the pressure in the cylinder. Specifications of the test engine are located in Appendix A. The angle of spark firing and burn duration angle were not explicitly known. Reasonable estimates for the timing were made from prior experiences and by visual approximation.

The model was fairly accurate in its cylinder pressure predictions. It was tested with engine pressure data at 2000 rpm full load (FL), 2000 rpm half load (HL), 4000 rpm FL, and 4000 rpm HL. The first figure in Appendix B plots the actual and model data at 4000 rpm FL. Of the four trials, the model deviated the most at 4000 rpm FL. The differences of all four trials between the model data and the actual data are shown in the second figure of Appendix B. Differences were greatest at higher rpm and greater load.

Under full load, the inlet pressure was about 1 atm and 0.5 atm at half load. The spark timing was changed from model to model. Generally, an increase in engine speed was accompanied by earlier spark firing and longer combustion duration. An increase in inlet pressure meant that the spark fired later, and the burn duration was shorter. The exact values of the inlet pressure in kilopascals (kPa) and the spark timing used in each model are given in Table 1.

The model results were compared to actual engine data. Differences in P_{spark}, $\theta_{peak\ pressure}$, P_{peak} and P_{EVO} were recorded and analyzed. The accuracy of P_{spark} is a measure of the performance of the compression model. Discrepancies in $\theta_{peak\ pressure}$ and P_{peak} gauge the model of combustion while P_{EVO} shows how well the expansion model described the expansion process. Tables 2 through 5 show the performance of the model under different engine conditions.

The compression model produced pressure values slightly greater than the actual values. The percent error of P_{spark} was around 10%. In an actual engine, some energy is lost during the compression process. Because the energy loss was not taken into account in the model, the pressure predictions at the spark firing angle were larger than the actual pressure values.

In some trials, the predicted angle of peak pressure in the model was less than the actual angle. Because the encoder on the engine that records the crank angle is sometimes faulty at high engine speed, the actual pressure data may be slightly inaccurate. The peak pressure was usually accurate but was at times lower than the actual value. These variations were, in part, due to the uncertainties in the angle spark firing and burn duration.

The cylinder pressure at EVO was the most variable. The model incorporated a representation for blowdown, which caused a visible drop off at EVO in the plots of the model, but the actual data did not show such a large decrease. The differences were more pronounced in the trials at higher engine speeds. These discrepancies in pressure during combustion and expansion were partially due to the mechanics of the pressure transducer. The transducer does not work well at high temperatures or at quickly changing temperatures. Because of the rapidly changing conditions in the cylinder, it may not have recorded all the changes in pressure during blowdown.

Another reason for the differences is the inaccuracy of the function that describes the area of the valve opening.

Trial	P_{inlet} (kPa)	θ_{spark}	$\Delta\theta$
2000 HL	63.7	18° BTC	71°
2000 FL	102.2	10° BTC	65°
4000 HL	45.0	27° BTC	75°
4000 FL	100.2	23° BTC	78°

Table 1. Model inputs. BTC stands for "before top center."

	Actual	Simulation	Difference
P_{spark} (kPa)	890	975	9.5%
$\theta_{peak\ pressure}$	16° ATC	16° ATC	0°
P_{peak}	3050	3050	0%
P_{EVO}	268	253	-5.6%

Table 2. Analysis of 2000 rpm HL. ATC stands for "after top center."

	Actual	Simulation	Difference
P_{spark} (kPa)	1730	1970	13.9%
$\theta_{peak\ pressure}$	22° ATC	22° ATC	0°
P_{peak}	3780	3780	0%
P_{EVO}	464	426	-8.2%

Table 3. Analysis of 2000 rpm FL. ATC stands for "after top center."

	Actual	Simulation	Difference
P_{spark} (kPa)	546	494	-9.5%
$\theta_{peak\ pressure}$	11° ATC	8° ATC	-3°
P_{peak}	2720	2710	-0.3%
P_{EVO}	221	164	-25.8%

Table 4. Analysis of 4000 rpm HL. ATC stands for "after top center."

	Actual	Simulation	Difference
P_{spark} (kPa)	1420	1280	-9.9%
$\theta_{peak\ pressure}$	20° ATC	13° ATC	-7°
P_{peak}	5320	5280	-0.8%
P_{EVO}	585	400	-31.6%

Table 5. Analysis of 4000 rpm FL. ATC stands for "after top center."

Load	P_{inlet} (kPa)
Quarter load	42
Half load	61.7
Three-quarters load	82
Full load	100

Table 6. Load inlet pressures.

Load	θ_{spark}	$\Delta\theta$	Error	Area Under Curve	Percent Error
1/4 load	13.5° BTC	56.4°	990	160000	0.62%
Half load	16.6° BTC	72.1°	958	247700	0.39%
3/4 load	16.0° BTC	80.5°	1110	296100	0.37%
Full load	13.1° BTC	84.6°	1700	352200	0.48%

Table 7. Optimal spark timings. BTC stands for "before top center."

Without actually measuring the area of the valve opening in the test engine, it is impossible to predict the changes in the size of the opening. Valves can be opened by more than one mechanism. The valves are opened at different rates, depending on the mechanism. Hence, the time it takes for the area to reach the maximum varies from engine to engine. The curve that predicted the area of the valve opening may have been incorrect for the test engine.

On the whole, the model predicted the pressure in the cylinder fairly accurately. The shape of the curve was well matched with the actual data. Pressure rose through compression, peaked during combustion and fell during expansion and blowdown. The standard deviation of the model was 1.33 for 2000 HL, 2.40 for 2000 FL, 3.13 for 4000 HL and 8.61 for 4000 FL. Considering that the pressure varies from 50 kPa to 5000 kPa, the model was very accurate.

Spark and Burn Duration Determination

The exact angle of spark firing and duration of combustion were not known. The engine cycles are completed extremely quickly and the spark timing varies slightly each cycle which made it difficult to accurately measure the spark timing. By varying spark firing and burn duration inputs of the model, the best fit curve was obtained that gave a prediction of the spark timing.

Data Fitting. The error of the model is calculated by an sum of the differences between the real and model pressure values:

$$error = \sqrt{\sum_{i=0}^{720}\left(P_{model}\left(\theta_i\right) - P_{actual}\left(\theta_i\right)\right)^2} \qquad (9)$$

Data was sampled at integral crank angles. The error was minimized to find optimal values for the angle of spark firing and burn duration. Pressure data came from a Renault production engine (see Appendix A for specifications). The pressure measurements were taken at 2000 rpm under quarter load, half load, three-quarters load and full load. The inlet pressures corresponding to the loads are given in Table 6.

The optimal angle of spark firing, θ_{spark}, and burn duration, $\Delta\theta$, for the sets of pressure data are given in Table 7. The error, approximate area of the actual pressure curve and the error as a percentage of the area are also given in the table.

The percent error of the model was approximately 0.5% for all the trials. Error increases with higher inlet pressures because of the greater pressures in the data.

Spark and Burn Duration Evaluation. The results of the spark firing and burn duration determination are shown in Table 7. Generally, the burn duration angle increases with load and, with the exception of the quarter load data, the angle spark firing becomes later with increasing load. The delay in spark firing as the inlet pressure increases is expected, however, the increase in burn duration goes against the predicted trend.

In the model, an increase in inlet pressure is reflected as a larger pressure at the spark and a greater mass of fuel in the cylinder. With more fuel to burn, the model lengthens the burn time. Burn duration is decreased only slightly by the increase in pressure at the spark.

In actuality, an increase in the initial pressure during combustion can greatly alter the way the fuel burns. Higher pressure in a limited volume means that the temperature in the cylinder is higher and that the fuel and air molecules are more reactive. The mixture burns faster, so the burn duration becomes shorter. The delayed spark firing takes advantage of the more reactive nature of the fuel to get more power out of the stroke.

The model does not take into account the increased temperature at the spark. The reactivity of the fuel is not a factor in the MLK combustion relation that is used by the model. Under higher inlet pressure, the gases in the cylinder are hotter throughout combustion and the combustion, which is an oxidation reaction, occurs faster. This increase in reaction rate is due to the increased kinetic energy so it takes a smaller amount of additional energy to reach the activation energy for the reaction.[6]

The MLK and similar burn fraction models were originally formulated from burn profile pictures. These relations were empirically determined by observing the photographs, which means that the models were not based on principles of thermodynamics. This fact caused the error in the model. An improvement would be to find a combustion model based on the work-energy theorem where pressure and temperature are related to the total energy of the system.

Summary

This model predicts the pressure of the gas in a cylinder by incorporating the isentropic gas process model, the McCuiston, Lavoie and Kauffman model and the gas flow through an orifice model. It predicts the pressure in the cylinder fairly accurately. The most significant weakness of the model is that is does not take the changes in fuel reactivity during combustion into account. However, the model is simple and can be used to give quick estimates of pressure for other models, such as oil film thickness.

Acknowledgments

This project would not have been possible without the help and advice of Professor John Heywood and Tian Tian at the Sloan Automotive Laboratory at MIT. Thanks also go to Bouke Noordzij and Peter Hinze for supplying engine data. The author is grateful to Mr. Don Hyatt for providing the use of the Thomas Jefferson High School Computer Systems lab. Lastly, the author thanks Dr. John Dell for his support and help through all the difficulties.

References

(1) Heywood, J. B. 1988. *Internal Combustion Fundamentals* (New York: McGraw-Hill, Inc.).

(2) Gyftopoulos, E. P., and G. P. Beretta. 1991. *Thermodynamics: Foundations and Applications* (New York: Macmillan Publishing Company).

(3) Cheung, H. M., and J. B. Heywood. 1993. "*Evaluation of a One-Zone Burn-Rate Analysis Procedure Using Production SI Engine Pressure Data*." SAE paper 932749.

(4) Amann, C. 1985. "*Cylinder-Pressure Measurement and Its Use in Engine Research*." SAE paper 852067.

(5) This equation was suggested by Professor John Heywood.

(6) Rossotti, H. 1993. *Fire* (New York: Oxford University Press).

Appendix A: Renault Engine Specifications

bore	83 mm
stroke	93 mm
compression ratio	9.8
ratio connecting rod to crank radius	3.2
IVO	5° BTC
IVC	43° ABC
EVO	52° BBC
EVC	1° ATC

Appendix B: Graphical Comparisons