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LINKING EVALUATION OF SUBJECTIVE TIRE TESTS ON THE ROAD WITH OBJECTIVELY MEASURED DATA

H. W. STUMPF*

Technikum Joanneum GmbH, A-8020 Graz, Alee PoststraBe 149, Austria

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ABSTRACT−Measurements of the initial values lead to an inverse and mathematically unprecisely formulated problem. A precise definition of an inverse problem is possible. It is to state a mathematical model of a physical process with clearly defined initial and exit values for the system behind the process. One can grasp the idea of an inverse problem by considering the tire as a copy of the objects of nature in a room with observations. Interpretation of nature is generally a result of an inverse problem. On one hand, the tire may be represented through the sensory organs and the nervous system as well as the experiences of the developer's existing apparatus of the projection of reality. On the other hand, it may be represented by a physical law or a model that can be confirmed or is to be refuted with the help of suitable measurements. During reconstruction of a measuring signal and the identification of a black box that can be assumed to be linear and causal, the tire becomes a first type Volterra integral equation of the convolution type. But measurements of the initial values are always fuzzy, the errors grow and the system behavior can no longer be forecasted. Thus, we have to deal with a chaotic system. This chaos produces fractals in a natural way. These are self-similar geometric structures. This self-similarity is clearly visible in the design.

KEY WORDS : Tire, Tire testing, Subjective tire testing, Objective tire testing, Fuzzy subset, Fractals

1. INTRODUCTION

The tire with its cord rubber structure is non-homogeneous and non-isotropic: Equation (1). The process of tire deflection leads to large deformations of the tire elements. It is well known that rubber is a viscoelastic material. The material Equation (2) for such a cord rubber structure does not only depend on temperature, displacement, and the strength, like stress- and strain-amplitudes, but also on the history of strength: Equation (3). The history of strength is a very important factor for a tire testing and tire calculations as shown in Table 1. By approaching the situation from this angle, we can only describe the tire behavior with test results, such as in Equations (4), (5), (6). To this end, we need models to show the reality of tire testing on the road which are based on either objectively measured data, subjectively performed tests, or laboratory tire tests as shown in Table 2 (Stumpf *et al*., 1988; Stumpf, 1997).

2. PREDICTABILITY

Usual engineering work succeeds top-down and is fully determinated. A totally different procedure for estimation of tire behavior a priori or better understanding of test results a posteriori is caused by the hermeneutic thinking process. This is a holistic approach to the world of physical objects and states. Inexact decisions are implemented in the modeling process which is part of constructive realism, and constructive realism leads to a hermeneutic thinking process. Furthermore, traditional technical science acts in the transcendental thinking process. The transcendental thinking process is a part of determinism. Determinism is another word for idealism. Idealism is not a part of reality. On the other hand, it may be confirmed by a physical law or a model which can be confirmed or is to be refuted with the help of suitable measurements. It can be shown that the solutions are differential equations with boundary value problems that are not directly soluble in all generality. The question is an inverse and mathematically unprecisely formulated problem (Wolfersdorf, 1994).

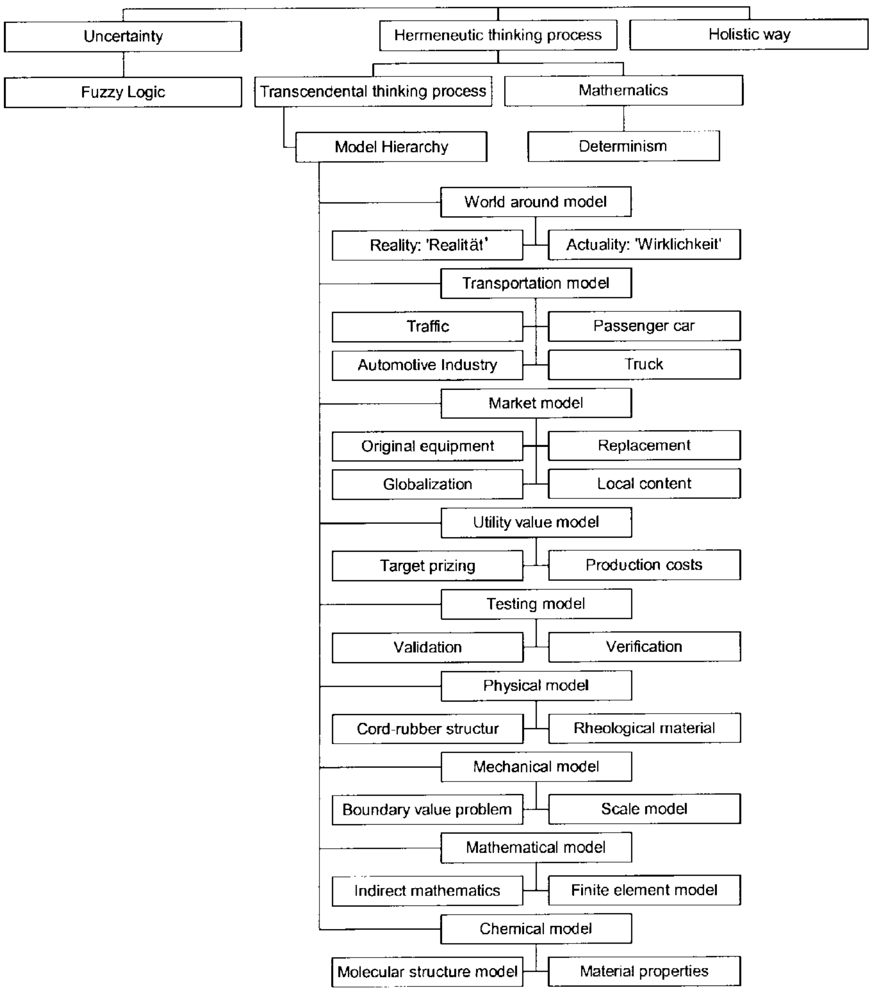
A precise definition of an inverse problem is possible. It is to state a mathematical model of a physical process with clearly defined initial and exit values for the system standing behind the process. One can grasp the idea of an inverse problem by considering the tire as a copy of the objects of nature in a room with observations. Interpretation of nature generally results as an inverse problem. Especially the tire can represent it through the sense

**Corresponding author.* e-mail: horst.stumpf@fh-joanneum.at

Table 1. The history of strength: solution as inverse mathematical problem.

Rubber		Toroidal pneumatic tire	
Material with fading memory, Stumpf (1998)		**Inisotropic, inhomogeneous material**	
$\mathbf{R}^{\mathrm{T}}\mathbf{T}\mathbf{R} = f(\mathbf{C}) + \mathop{\mathcal{F}}_{s=0}^{\infty} \mathrm{F}(\mathbf{G}^{*}(s);\ \mathbf{C})$	(2)	Boundary value problem $\mathbf{T}_{\mathbf{R}} = h(\mathbf{F}, \chi)$; $\chi = x(\chi)$; $\mathbf{F} = \nabla\ \chi(\chi)$	(1)
R = Rotation tensor; **T** = Stress tensor; **G** = History of strength; **C** = Right Cauchy-Green tensor		**F** = Strength; χ = Place coordinates	
Incompressible material			
$(\mathbf{G}(\mathbf{s}) + \mathbf{1}) = \mathbf{1}$	(3)		
Aging			
$\ln\ \tau = f(1/\Theta)$	(4)		
τ = Time of strength; Θ = Temperature			
Oxydative strength			
$\Delta p_{I,t} = f(\Theta,\ \tau,\ h\ \Delta p_0,\ \mu)$	(5)		
h = Thickness; Δp_0 = Inner pressure; μ = Permeability			
Tearing energy			
$\mathrm{T_F} = -\{\partial W/\partial A\}_h$	(6)		
$\mathrm{T_F}$ = Tearing energy; W = Deformation energy			

Table 2. Model hierarchy.



organs and the nervous system as well as the experiences of the developer's existing apparatus of the projection of reality. During reconstruction of a measuring signal and the identification of a black box that can be assumed to be linear and causal, the tire becomes a first type Volterra integral equation of the convolution type.

$$y(t)=\int_0^1 f(t-s)x(s)ds \tag{7}$$

The problems of reconstructing a measuring signal and identifying a black box can be assumed to be linear and causal. The mathematical model of the black box becomes described by Equation (7). Equation (7) can be solved formally with the help of the Laplace transformation. In theory, partial differential equations executed correctly put classic problems of mathematical physics ingeniously as direct problems and their differential reversals as indirect problems. In the more narrow sense, the problems lies in the determination of coefficients and constitute differential equations. Therefore, the material quality from certain functions of their solutions is vital.

The boundary value problem of an elliptical differential equation is given in Equation (8).

$$\begin{aligned} &\mathrm{div}[a(x)\mathrm{grad}u] + b(x)u = f(x) \\ &x \in G \leq \boldsymbol{R}^n \\ &u(x) = g(x) \mapsto \text{Boundary condition} \\ &x \in \boldsymbol{S} \mapsto \text{Boundary of } G \end{aligned} \tag{8}$$

G describes a limited area at the n-dimensional room $\boldsymbol{R}^n$. A fundamental inverse problem to it is the determination of the coefficients $a(x)$ and $b(x)$. A further problem creates the reconstruction the boundary value g on $\boldsymbol{S}$ or a part of it, if values of u are given. Thus, the calculation of the tire presents fundamentally an inverse problem, with the exception of a few special cases. These are bad prognoses for finite element calculations.

3. THE FUZZY SUBSET

The fuzzy set theory has established itself in tire development. In the future, its usage will increase even more (Lux, 1993). It will be used more as direct and indirect mathematics than as a tool. That will occur, should conventional decision criteria like yes-no, or 0-1, not deliver the desired results, e.g. about the correlation of objective results with subjective judgments, automated production machines are automated, etc. We have to do it by treating the tire as a fuzzy problem.

Let us assume a set E with an element x and a subset A of x: Equation (9).

$$\begin{aligned} &A \subseteq E \\ &x \in A \rightarrow x \in E \end{aligned} \tag{9}$$

The characteristic function μ becomes described by Equation (10).

$$\begin{aligned} &\mu: E \rightarrow [0,1] \\ &\mu(x)=1 \text{ if } x \in A\text{; } 0 \text{ if } x \notin A \\ &A = \int_{x \in E} (\mu(x)|x) \end{aligned} \tag{10}$$

Fuzzy subset $\tilde{A}$ of E is a set of ordered pairs: Equation (11).

$$(x,\ \mu_A(x)),\ \forall x \in E \tag{11}$$

Membership function becomes described by Equation (12).

$$x \underset{\tilde{A}}{\overset{}{\underset{\mu}{\Rightarrow}}} M \tag{12}$$

The union of two sets becomes described by Equation (13).

$$\begin{aligned} &\tilde{A} \subseteq E;\ \tilde{B} \subseteq E \\ &\tilde{A} \cup \tilde{B}=\int_{x \in E} (\max(\mu_A(x),\ \mu_B(x))|x) \end{aligned} \tag{13}$$

Intersection of two sets: Equation (14).

$$\tilde{A} \cap \tilde{B}=\int (\min(\mu_A(x),\ \mu_B(x))|x) \tag{14}$$

Complement of set: Equation (15).

$$\bar{A}=\int_{x \in E} (1-\mu_A(x)|x) \tag{15}$$

A fuzzy relation R between X and Y is a fuzzy subset of the Cartesian product X x Y by a matrix T_R: Equation (16).

$$\begin{aligned} &\tilde{R}=\int_{(x,y) \in X \times Y} (\mu(x,\ y)|(x,\ y)) \\ &X=\{x_1,\ x_2,\ \ldots\ x_n\} \\ &Y=\{y_1,\ y_2,\ \ldots\ y_n\} \end{aligned} \tag{16}$$

Fuzzy relations R on X x Y and S on X x Z: Equation (17).

$$\begin{aligned} &\tilde{R} \times \tilde{S}=\int_{y \in Y,(x,z) \in X,Z} \max\ \min(\mu(x,\ y),\ \mu(y,\ z))|(x,\ z) \\ &\tilde{R} \times \tilde{S}=\int_{y \in Y,(x,z) \in X,Z} \max(\mu(x,\ y) \times (y,\ z))|(x,\ z) \\ &T_{R \times S}=T_R \times T_S \end{aligned} \tag{17}$$

4. OBJECTIVE MEASUREMENT CRITERION

Static, quasi-static and dynamic measurements on tires were performed in the Automotive Engineering lab, Graz: Equation (18).

$$F_y=c_{a0} \times \bar{y}=c' \times \alpha_{eff} \times F_z$$

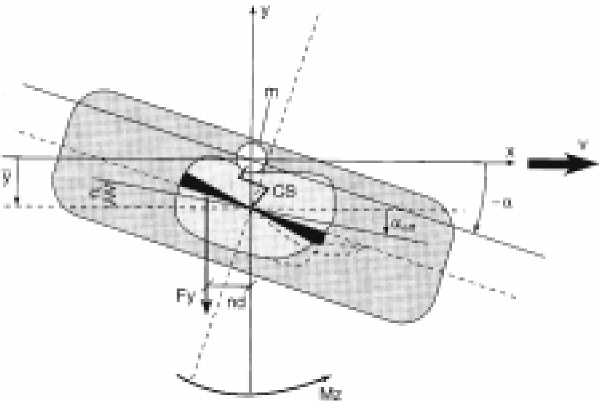


Figure 1. Wheel model.

$$\alpha_{eff}=\alpha-\left(\frac{\dot{\bar{y}}}{v}\right) \tag{18}$$

v ... Speed

The model of a deformed tire is shown in Figure 1 (Stumpf, 1997).

The definition of measured data in regard to stationary road tests such as circle tests is described by Equation (19).

α ... Slip angle; α_{eff} ... Real slip angle
F_z ... Tire load; F_y ... Side force
c_s ... Axial stiffness of the wheel (19)

$$F_y{}'=\frac{\partial F}{\partial \alpha}\bigg|_{\alpha=0} \text{ ... Gradient of side force}$$

$$c'=\frac{F_y{}'}{F_z} \text{ ... Cornering stiffness coefficient}$$

Non stationary road tests such as a steering response test, J-test, slalom test and lane change test, are described by Equation (20).

M_z ... Self aligning torque
$\bar{y}$ ... Axial distortion of tire
c_a ... Dynamic axial stiffness
c_{a0} ... Quasi static axial stiffness

$$M_z{}'=\frac{\partial M_z}{\partial \alpha}\bigg|\alpha=0 \text{ ... Steering resistance}$$

$$n_d=\frac{M_z{}'}{F_y{}'}\bigg|\alpha=0 \text{ ... Dynamic caster} \tag{20}$$

$$l_e=\frac{c'\cdot F_y}{c_{a0}} \text{ ... Free rolling offset}$$

c_y ... Axial stiffness of tread pattern
LS ... Load sensitivity
$1/\gamma$ ... Bending stiffness
LI ... Steering impulse

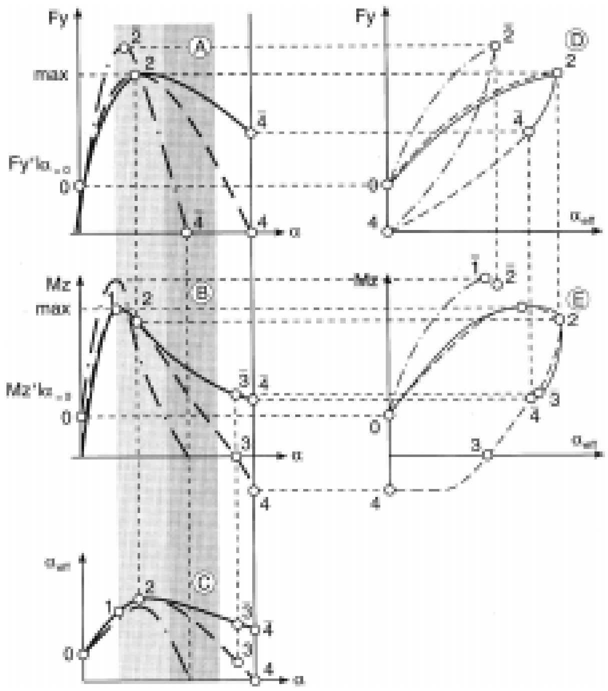


Figure 2. Effective slip angle.

ΔF ... Variation of footprint surface

The effective slip angle can be seen in Figure 2.

If one combines the diagrams A and C, diagram D emerges. Diagram E shows the true self alignment torque.

5. SUBJECTIVE TEST DISCIPLINE

A reduced data set is necessary to create useful correlations. However, in the case of some single elements, the values of the rear axle RA and the front axle FA should be taken into consideration as shown in Table 3.

In the case of some subjective or objective categories, we can summarize these categories as a single expression. Every element has only one membership factor. This membership factor is according to the max-min-principle of fuzzy-set-theory.

In order to categorize the standardized ‘Gutlage’-diagram, we examine the criteria ‘position’ and ‘gradient’: Equation (21).

$$\frac{c_{a0}}{F_z}\cdot c' \text{ ... Position} \tag{21}$$

Angle of distance FA-RA to c'-axle ... Gradient

The criteria total ‘Gutlage’ is composed of those criteria characterized in Table 4.

The marking of a subjective vehicle performance test

Table 3. Correlation.

Subjective test	Objective criterion
Steering response $\beta <<$	c' (FA) c_{a0} (FA) M_z' (FA) c_y (FA) $1/\gamma$ (FA)
Slalom test	LS (FA+RA) c_{a0} (FA+RA) LI (FA) $\alpha\|_{M_z=max}$ (FA) $1/\gamma$ (FA+RA)
Curve behavior	ΔA (FA+RA) $\|c'\|,\|M_{zmax}\|$ (FA+RA)$\|_{\text{Gutlage}}$ (Gradient)(FA+RA)$\|_{\text{Gutlage}}$ LS (FA+RA) $1/\gamma$ (FA+RA)
Handling characteristic	ΔA (FA+RA) (Diagram(FA+RA))$\|_{\text{Gutlage}}$ $\alpha\|_{M_z=max}$ (FA) $1/\gamma$ (FA+RA)
Test condition	Front axle..FA; Rear axle..RA

has to be examined as membership to category of that mark with the certain membership factor. These categories of marks extend in step from 1 to 10. The size of this factor is to be estimate with regard to a single mark from the specific vehicle performance criteria.

Table 4. Categories in the ‘Gutlage’-diagram.

Total	Position	Gradient
G: Large	G G M	G M G
M: Middle	M G K	M K G
K: Low	K M K	K K M

6. ‘GUTLAGE’-DIAGRAM

In a lane-change test the effect of tires on steering conditions and damped free vibration response or post steering wheel turn respectively may be seen as shown in Figure 3.

7. FRACTALS

Measurements of initial values are always fuzzy, the errors grow, and the system behavior can no longer be

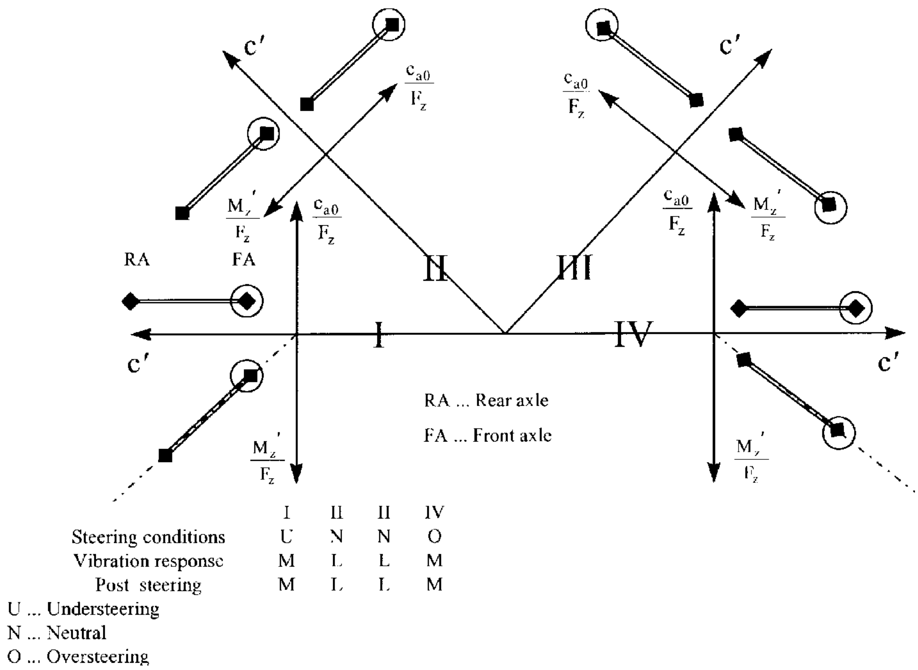


Figure 3. Lane change test and ‘Gutlage’-diagram.

forecasted. Thus, we have to deal with a chaotic system (Maletsky, 1992).

A pneumatic tire is made up of different components namely, tread, side wall, carcass, belt, bandage, bead, liner etc. Different types of rubber compounds each with different chemical recipe and physical characteristics are used in the preparation of different tire components (Haridas, 1994). Of the general requirements of a pneumatic tire, the prime concern is regarding tire handling parameters-namely the load carrying capacity (FA and RA) and velocity.

Let us consider a coordinate system x_1, x_2 ... x_n called order parameters. Let c_1, c_2 ... c_k be another set of parameters which represents forces on the system.

The total energy E of the system: Equation (22)

$$E=KE+PE$$
$$KE=1/2 \cdot \mu_{ij} \cdot \dot{x}_i \cdot \dot{x}_j \geq 0 \quad \text{... Kinetic energy} \qquad (22)$$
$$PE=V(x;c) \quad \text{... Potential energy}$$
$$\dot{x}_i \quad \text{... Generalized velocity}$$

The Taylor series expansion of the ideal system has the form: Equation (23)

$$V(x;F)=1/2 \cdot V_{ij}(F) \cdot x_i \cdot x_j + \text{higher order forms} \qquad (23)$$

When the tire is subjected to a dynamic loading as the case in actual service conditions, we introduce another perturbation factor, ε.

Average kinetic energy added to the system due to movement: Equation (24)

$$\Delta E=V^{(i)}-V^{(j)} \qquad (24)$$
$$i, j=0, 1, 2, \ldots x_0; \; i \neq j$$

The use of catastrophy theory allows the reduction in force to be put into a quantitative form: Equation (25)

$$F_C=F_P \cdot k \cdot |\varepsilon| \cdot P$$
$$k \quad \text{... Positive constant}$$
$$P \quad \text{... Positive rational fraction} \qquad (25)$$
$$\varepsilon \quad \text{... Imperfection factor}$$
$$F_C \quad \text{... Control load}$$

Two tires with identical potential function $V(x; f; \varepsilon)$ may have different kinetic energy functions. Their behavior under static load may be identical, but under dynamic conditions they may differ. Mathematically, this explains the premature differences of similar tires under different service conditions.

8. CONCLUSIONS

In tire construction, chaos models of various systems can be worked out for various processing states, such as compound mixing, extrusion of components, the analysis of physical and dynamic properties of tires and design, etc. Studies in this area will lead to the development of governing equations and subsequent models that give more precise and clear understanding of the tire system.

This chaos produces fractals in a natural way. Fractals are self-similar geometric structures This self-similarity is clearly visible in the design.

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APPENDIX

OTHER OBJECTIVE MEASUREMENT CRITERIA

Axial stiffness of tread pattern: Equation (26)

$$c_y=\frac{F_y'}{h^2 \cdot b_r(1-\gamma \cdot h \cdot F_y')} \qquad (26)$$

h ... Half length of footprint

Bending stiffness: Equation (27)

$$\frac{1}{\gamma}=F_y' \cdot \frac{\frac{b^3}{b_r}+2 \cdot h \cdot \left(\frac{M_z'}{F_y'}-\frac{h}{3}\right)}{\left(\frac{M_z'}{F_y'}-\frac{h}{3}\right)} \qquad (27)$$

b_r ... Reduced with of footprint

Load sensitivity: Equation (28)

$$LS = \left.\frac{F_{y|n} - F_{y|0.8 \cdot n}}{F_{z|n} - F_{z|0.8 \cdot n}}\right| \alpha = 1^{o} \quad (28)$$

Steering impulse: Equation (29)

$$LI = \int F_y(t) \cdot dt \quad (29)$$

Handling coefficient: Equation (30)

$$TB = T_{\psi max} \times \beta_{St} = min$$
$$HC = LI \times c' = max \quad (30)$$

Cornering coefficient: Equation (31)

$$CC = \left.\frac{F_y}{F_z}\right| \alpha = 1^{o} \quad (31)$$

Aligning torque coefficient: Equation (32)

$$ATQ = \left.\frac{M_z}{F_z}\right| \alpha = 1^{o} \quad (32)$$

Load transfer sensitivity: Equation (33)

$$LTS = \left.\frac{\Delta F_y \cdot F_z}{(\Delta F_z)^2}\right| \alpha = 4^{o} \quad (33)$$

Variation of footprint: Equation (34)

$$\Delta A = \left| A_S^{FA} - A_S^{RA} \right| \quad (34)$$

$A_S^{FA,RA}$ ... Effective footprint area